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NO. 447

NUMERICAL SIMULATION OF THE AIR BLAST RESPONSE OF TAPERED CANTILEVER BEAMS (U)

by

G.V. Price

PROJECT NO. 97-80-01

November 1977

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ABSTRACT

A numerical procedure is developed to predict the elastic response of variable cross-section cantilever beams when subjected to a transient air blast load. Computed natural frequencies and transient strains were in reasonable agreement with experimentally obtained values.

(U)

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NUMERICAL SIMULATION OF THE AIR BLAST RESPONSE OF TAPERED CANTILEVER BEAMS (U)

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1. Introduction

The Defence Research Establishment Suffield (DRES), in support of Canadian Forces (Maritime) requirements, is conducting a series of tests to determine the ability of certain antenna designs to withstand blast overpressures of various intensities. Two of the antennas to be evaluated are a 35 ft fibreglass Whip Antenna and a 23 ft UHF Polemast Antenna. The objective of this study is to develop a numerical procedure to predict the time response of antennas of the above type when subjected to a transient air blast load.

The numerical procedure begins with the Bernoulli-Euler equation for a tapered cantilever beam subjected to a transient distributed force. This equation is based on linear elastic theory and assumes small beam deflections [1]. The normal modes and natural frequencies of the beam are determined by solving the difference equations for free vibration using successive relaxation, Rayleigh quotient and Gram-Schmidt orthogonalization numerical techniques [2,3]. The forced vibration solution is determined using normal mode coordinates and Laplace transforms [4]. In this calculation,

the transient air blast load is computed using the classical gas dynamics equations describing air shock waves [5,6,7] and recent empirical drag loading relations developed at DRES [8,9].

The Bernoulli-Euler equation for a vibrating beam of length L may be written in the form [1]

$$\rho A \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) = w, \; 0 \le x \le L, \tag{1}$$

where x is distance from the base of the beam, t is time, $y(x,t)$ is the beam deflection in the transverse direction, ρ is the beam density, $A(x)$ is the beam cross-sectional area, E is Young's modulus for the beam material, $I(x)$ is the second moment of the cross-sectional area with respect to the neutral axis, and $w(x,t)$ is the load per unit length on the beam in the transverse direction. The discussion which follows considers a numerical solution to the above equation at N prescribed points

$$x_i = i\,\Delta x, \quad \Delta x = LN^{-1}, \quad i = 1, \ldots, N, \tag{2}$$

subject to specified initial conditions and boundary conditions.

2. Free Vibration of a Tapered Cantilever Beam

The normal modes and natural frequencies are obtained by solving equation (1) with w=0. Assuming a sinusoidal time response, the free vibration solution may be written in the form

$$y(x,t) = Y^{(r)}(x)e^{i\omega_r t}, \tag{3}$$

where ω_r is the r'th circular frequency of the time response, $Y^{(r)}(x)$ is the normal mode corresponding to ω_r, and i is $\sqrt{-1}$. With the results (2) and (3), equation (1) with w=0 reduces to the form

$$\rho A_i \, \omega_r^2 \, Y_i^{(r)} = \frac{d^2}{dx^2}\left(EI_i \frac{d^2}{dx^2} Y_i^{(r)}\right) \tag{4}$$

at point x_i. Expanding the derivative in (4) using central finite differences, the difference equation for free vibration becomes

$$\begin{aligned} I_{i-1} Y_{i-2}^{(r)} - 2(I_{i-1} + I_i)Y_{i-1}^{(r)} + (I_{i-1} + 4I_i + I_{i+1} - \rho A_i \omega_r^2 \Delta x^4 E^{-1})Y_i^{(r)} \\ -2(I_i + I_{i+1})Y_{i+1}^{(r)} + I_{i+1} Y_{i+2}^{(r)} = 0, \quad i = 1, 2, \ldots, N. \end{aligned} \tag{5}$$

The four unknowns $Y_{-1}^{(r)}$, $Y_0^{(r)}$, $Y_{N+1}^{(r)}$ and $Y_{N+2}^{(r)}$ are eliminated from this system of equations through the application of boundary conditions. Several possibilities are considered below.

(a) clamped at x = 0, zero displacement and slope:

$$\left.\begin{aligned} y(0) &= 0 , & Y_0^{(r)} &= 0 ; \\ \left.\frac{\partial y}{\partial x}\right|_0 &= 0 , & Y_{-1}^{(r)} &= Y_1^{(r)} . \end{aligned}\right\} \tag{6}$$

(b) free at x = L, zero moment and shear:

$$\left.\begin{aligned} \left.\frac{\partial^2 y}{\partial x^2}\right|_L &= 0 , & Y_{N+1}^{(r)} &= 2Y_N^{(r)} - Y_{N-1}^{(r)} ; \\ \left.\frac{\partial^3 y}{\partial x^3}\right|_L &= 0 , & Y_{N+2}^{(r)} &= 4Y_N^{(r)} - 4Y_{N-1}^{(r)} + Y_{N-2}^{(r)} . \end{aligned}\right\} \tag{7}$$

(c) <u>pinned at x = 0 and x = IΔx</u>, zero displacement and moment at x = 0, and zero displacement at x = IΔx:

$$\left.\begin{aligned} y(o) &= \left.\frac{\partial^2 y}{\partial x^2}\right|_0 = 0 , \qquad Y_0^{(r)} = 0,\ Y_{-1}^{(r)} = -\,Y_1^{(r)} ; \\ y(I\Delta x) &= 0 \qquad\qquad , \qquad Y_I^{(r)} = 0 . \end{aligned}\right\} \tag{8}$$

With a suitable combination of the boundary conditions (6) to (8), equation (5) becomes an eigenvalue problem for an n-rowed square matrix. In matrix form, the eigenvalue problem may be written in the form

$$\omega_r^2\ \beta\ \underset{\sim}{Y}^{(r)} = \alpha\ \underset{\sim}{Y}^{(r)} , \tag{9}$$

where α is a five-diagonal matrix which depends on the coefficients I_i, i=1,...N, and β is a diagonal matrix for which the i'th diagonal element is the value $\rho A_i \Delta x^4 E^{-1}$. If the frequencies ω_r are all distinct, it may be readily verified that the normal modes are orthogonal with respect to the matrix β, according to

$$\left.\begin{aligned} \underset{\sim}{Y}^{(r)}\ \beta\ \underset{\sim}{Y}^{(s)} &= \delta_{rs}\ M_r , \\ \delta_{rs} &= \begin{cases} 0 , & r \neq s , \\ 1 , & r = s . \end{cases} \end{aligned}\right\} \tag{10}$$

A numerical solution procedure for the eigenvalue problem (9) will now be considered. The procedure begins with an initial guess for the smallest circular frequency ω_1. The corresponding normal mode is obtained by successive relaxation [2,3] according to the iteration equation

$$\underset{(n+1 \text{ guess})}{Y_i^{(1)}} = (\omega_1^2 \beta_{11} - \alpha_{11})^{-1} \sum_{\substack{j=1 \\ (j \neq i)}}^{N} \alpha_{ij} \underset{(n \text{ guess})}{Y_j^{(1)}} . \tag{11}$$

This iteration is inherently unstable since the system of equations is homogeneous. However, convergence to a stationary $\underset{\sim}{Y}^{(1)}$ is achieved by normalizing $\underset{\sim}{Y}^{(1)}$ after each complete repetition of the iteration. An improved estimate for ω_1 is then obtained by the Rayleigh quotient [2,3] technique.

$$\underset{(\text{new estimate})}{\omega_1} = \frac{\sum_{i=1}^{N} Y_i^{(1)} \beta_{ii}^{-1} \sum_{j=1}^{N} \alpha_{ij} Y_j^{(1)}}{\sum_{i=1}^{N} Y_i^{(1)} Y_i^{(1)}} . \tag{12}$$

The complete iteration is repeated until a stationary value for the circular frequency ω_1 is obtained, at which point the corresponding normal mode, in normalized form, will be $\underset{\sim}{Y}^{(1)}$.

Higher natural frequencies ω_r and normal modes $Y^{(r)}$ are obtained using the above procedure with but one modification: between repetitions of the successive relaxation iteration (equation 11), the latest guess $\underset{\sim}{Y}_j^{(r)*}$ is modified to extract any component in the direction of previously evaluated normal modes $\underset{\sim}{Y}^{(s)}$, $s<r$. This operation is termed Gram-Schmidt orthogonalization [2,3], and is based on the orthogonality property (10).

$$\underset{(\text{new guess})}{\underset{\sim}{Y}^{(r)}} = \underset{\sim}{Y}^{(r)*} - \sum_{s=1}^{r-1} \underset{\sim}{Y}^{(s)} \left\{ \frac{\langle \underset{\sim}{Y}^{(r)*}, \underset{\sim}{Y}^{(s)} \rangle}{\langle \underset{\sim}{Y}^{(s)}, \underset{\sim}{Y}^{(s)} \rangle} \right\} . \tag{13}$$

In this equation, the inner product, denoted $<\underset{\sim}{u}, \underset{\sim}{v}>$ for two arbitrary vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$, is weighted with respect to the matrix β according to

$$<\underset{\sim}{u}, \underset{\sim}{v}> = \underset{\sim}{u}^T \beta \underset{\sim}{v} , \qquad (14)$$

where superscript T denotes the transpose of the indicated vector. A computer program which determines the natural frequencies and normal modes according to the techniques described above is given in the Appendix.

3. Forced Vibration of a Tapered Cantilever Beam

The forced vibration solution begins with an expansion of the displacement vector $\underset{\sim}{y}$ in terms of normal mode coordinates $\underset{\sim}{\eta}$, according to

$$\left.\begin{aligned} \underset{\sim}{y} &= \sum_{r=1}^{N} \eta_r \underset{\sim}{Y}^{(r)} , \\ &\text{or} \\ \underset{\sim}{y} &= \Phi \underset{\sim}{\eta} , \end{aligned}\right\} \qquad (15)$$

where Φ is a matrix formed by the normal modes.

$$\Phi = \begin{pmatrix} Y_1^{(1)} & \dots & Y_1^{(N)} \\ \vdots & \vdots\vdots\vdots & \vdots \\ Y_N^{(1)} & \dots & Y_N^{(N)} \end{pmatrix} . \qquad (16)$$

Using the general techniques outlined in [4], substituting (15) into (1) and pre-multiplying the resulting equation by Φ^T, the governing equation becomes

$$\Phi^T \beta \Phi \ddot{\underset{\sim}{\eta}} + \Phi^T \alpha \Phi \underset{\sim}{\eta} = \Phi^T \underset{\sim}{w} , \tag{17}$$

where $\ddot{}$ denotes the time derivative d^2/dt^2. The r'th equation in the system of equations (17) is

$$\underset{\sim}{\gamma}^{(r)^T} \beta \underset{\sim}{\gamma}^{(r)} \ddot{\eta}_r + \underset{\sim}{\gamma}^{(r)^T} \alpha \underset{\sim}{\gamma}^{(r)} \eta_r = \underset{\sim}{\gamma}^{(r)^T} \underset{\sim}{w} . \tag{18}$$

Since the normal modes satisfy equations (9) and (10), the above result simplifies to the form

$$\ddot{\eta}_r + \omega_r^2 \eta_r = \frac{\underset{\sim}{\gamma}^{(r)^T} \underset{\sim}{w}(t)}{M_r} . \tag{19}$$

The normal mode coordinates $\eta_r, r=1,\ldots N$ are obtained by solving equation (19) using Laplace transforms and the convolution theorem. With specified initial conditions in the form

$$\left.\begin{aligned} y(x,0) &= f(x) , \\ \dot{y}(x,0) &= g(x) , \end{aligned}\right\} \tag{20}$$

The solution to equation (19) becomes

$$\begin{aligned} \eta_r(t) = M_r^{-1} \underset{\sim}{\gamma}^{(r)^T} &\Big(\beta \underset{\sim}{f} \cos\omega_r t + \beta \underset{\sim}{g}\, \omega_r^{-1} \sin\omega_r t \\ &+ \omega_r^{-1} \int_0^t \underset{\sim}{w}(\tau) \sin\omega_r(t-\tau) d\tau\Big) . \end{aligned} \tag{21}$$

The parameter M_r appearing in this equation is evaluated according to equation (10).

$$M_r = \underset{\sim}{\gamma}^{(r)^T} \beta \underset{\sim}{\gamma}^{(r)} . \tag{22}$$

Combining the results (21) and (15), the beam displacement at position x_i becomes

$$y_i(t) = \sum_{r=1}^{N} Y_i^{(r)} M_r^{-1} \left\{ \cos\omega_r t \left(\sum_{j=1}^{N} Y_j^{(r)} \rho A_j f_j \right) \right.$$

$$+ \omega_r^{-1} \sin\omega_r t \left(\sum_{j=1}^{N} Y_j^{(r)} \rho A_j g_j \right) \tag{23}$$

$$\left. + \omega_r^{-1} \int_0^t \left(\sum_{j=1}^{N} Y_j^{(r)} w_j(\tau) \right) \sin\omega_r (t-\tau) d\tau \right\},$$

where
$$M_r = \sum_{j=1}^{N} Y_j^{(r)} \rho A_j Y_j^{(r)} . \tag{24}$$

The time integral in (23) may be readily evaluated by numerical integration using a time step Δt of specified duration.

A computer program which determines the forced vibration solution according to the techniques described above is given in the Appendix.

4. Transient Drag on Circular Cylinders from Air Blast Waves

The accuracy of the forced vibration solution is strongly dependent on the procedure used to calculate the air blast load on the tapered cantilever beam. The transient air blast load on circular cylinders has been investigated extensively at DRES over the period 1968 to 1975. The discussion which follows is based on the more recent DRES reviews of empirical relations for the prediction of drag loading [8,9] and on the classical gas dynamics equations describing air shock waves [5,6,7].

The air blast load arising from the interaction of a blast wave with a circular cylinder may be expressed in the form

$$F = P^*A + C_D \, q_I \, A \, , \tag{25}$$

where A is the projected area of the cylinder, P* is an empirical pressure parameter which accounts for the load on the cylinder over the short period of time during which the blast wave diffracts around and engulfs the cylinder, C_D is an empirical aerodynamic drag coefficient, and q_I is the impact pressure (instantaneous difference between the local stagnation and static pressures).

The following empirical relations have been determined at DRES for the diffraction pressure P* and the drag coefficient C_D [8,9]:

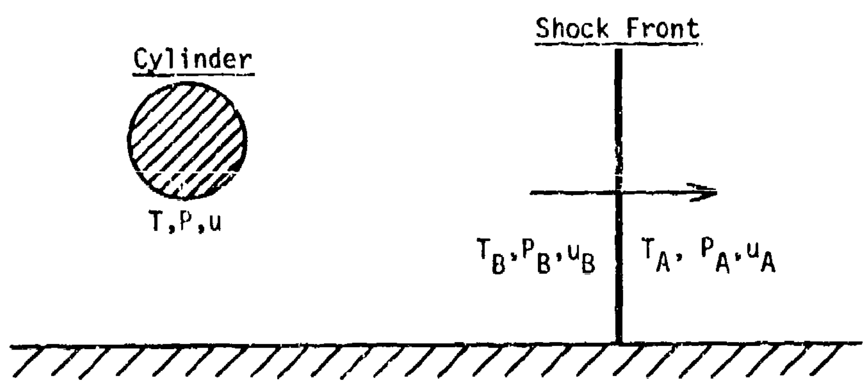
$$P^* = \left\{ \begin{array}{ll} 1.6 \, t_T^{-1} (p_O t_T)^{1.13} \, , & 0 \le t \le 5t_T \, , \\ 0 \, , & t > 5t_T \, , \end{array} \right\} \tag{26}$$

$$C_D = \left\{ \begin{array}{ll} 0.7 \, , & M \ge 0.48, \; Re \ge 3 \times 10^5 \, , \\ 0.6 \, , & M < 0.48, \; Re \ge 3 \times 10^5 \, , \\ 1.2 \, , & M < 0.48, \; Re < 3 \times 10^5 \, . \end{array} \right\} \tag{27}$$

In the above equations, p_O is the peak overpressure of the blast wave, t_T is the transit time required by the blast wave to completely engulf the cylinder, M is the instantaneous Mach number of the flow incident on the cylinder, and Re is the instantaneous Reynolds number of the flow incident on the cylinder.

To complete the transient drag calculation, it remains to evaluate the basic fluid properties of the flow behind the blast wave. The parameters to evaluate include q_I, t_T, M and Re. The evaluation of these parameters requires

a preliminary evaluation of the temperature T, speed of sound c, static pressure P, dynamic pressure q, fluid particle velocity u, and fluid viscosity ν both at the cylinder and immediately behind the shock front. To simplify the presentation, the following notation will be adopted. Variables which apply immediately in front of the shock front will be indicated by a subscript "A", variables which apply immediately behind the shock front will be indicated with a subscript "B", and variables which apply instantaneously at the cylinder itself will have no explicit subscript.



The pressure-time history of the ideal blast wave is most frequently described in terms of the modified Friedlander equation [7],

$$p(t) = p_o(1-t/t_d)e^{-\kappa t/t_d}, \quad 0 \le t \le t_d, \qquad (28)$$

where p(t) is the blast wave overpressure, t is the time measured from time of arrival of the blast wave, p_o is the peak overpressure, t_d is the positive phase duration, and κ is the exponential decay constant. The three parameters p_o, t_d and κ in this equation make it possible to match the following three experimental characteristics of the real blast wave: p_o, t_d and I_d, where I_d

is the positive phase impulse defined by

$$I_d = \int_0^{t_d} p(t)\, dt \quad . \tag{29}$$

The region immediately in front of the shock front is assumed to be in an undisturbed state (u_A=0) at a known pressure P_A and temperature T_A. Conditions immediately behind the shock front are determined from the Rankine-Hugoniot and other standard gas dynamics equations describing shock waves in air. Assuming a specific heat ratio γ = 1.4, and adopting the notation ζ to represent the pressure ratio P_B/P_A across the shock front, the fluid properties immediately behind the shock front may be written in the form

$$\left.\begin{aligned} P_B &= P_A + p_o \;, \\ T_B &= T_A\, \zeta\, (6 + \zeta)(1 + 6\zeta)^{-1} \;, \\ u_s &= (1.4\, R\, T_A)^{\frac{1}{2}}(1 + 6p_o/(7P_A))^{\frac{1}{2}} \;, \end{aligned}\right\} \tag{30}$$

where R is the gas constant for dry air, and u_s is the speed of the shock front. At the cylinder itself, conditions are determined by assuming isentropic flow in the region behind the shock front.

$$\left.\begin{aligned} P &= P_A + p(t) \;, \\ T &= T_B(P/P_B)^{.286} \;, \\ c &= (1.4\, RT)^{\frac{1}{2}} \;, \\ q &= 2.5\, p_o^{\,2}(p_o + 7P_A)^{-1}(p(t)/p_o)^2 \;, \\ \rho &= P(RT)^{-1} \;, \\ u &= (2q/\rho)^{-\frac{1}{2}} \;. \end{aligned}\right\} \tag{31}$$

With the basic fluid properties known behind the shock front and at the cylinder, it is possible to evaluate the four parameters q_I, t_T, M and Re appearing in the drag equations (25) to (27).

$$
\begin{aligned}
M &= u\, c^{-1}\,, \\
Re &= u\, D\, \nu^{-1}, \\
t_T &= D\, u_s^{-1}\,, \\
q_I &= q + .175\, PM^4 + .0175\, PM^6\,,
\end{aligned}
\qquad (32)
$$

where the kinematic viscosity ν is a known function of T and P (refer to [10] and [11] for considerations in this regard).

A computer program which determines the air blast load on circular cylinders according to the techniques described above is given in the Appendix.

5. Numerical Predictions and Comparison with Experiments

The numerical simulation model developed above was used to predict the time response of a 35 ft fibreglass Whip Antenna and a 23 ft UHF Polemast Antenna when subjected to a nominal 7.0 psi peak overpressure blast loading. The two antennas were subsequently exposed to air blast loading in Event Dice Throw, a 620 ton AN/FO free-field blast trial conducted by the United States Defense Nuclear Agency at the White Sands Missile Range in New Mexico on October 6, 1976, and strain data were recorded for five pairs of strain gauges measuring bending strain on each antenna. The theoretical predictions and experimental results for the Whip and Polemast Antennas have been examined in detail in references [12] and [13]. The brief discussion which follows considers only representative items from the detailed reports.

The structures of the antennas were represented in the computer model

in such a way as to simulate the mass and projected (normal to blast direction) cross-sectional area profiles of the prototype antennas. Photographs of the two antennas installed at the nominal 7.0 psi peak overpressure location for Event Dice Throw are shown in Figures 1 and 2. The physical features which describe the corresponding computer simulation of the antennas are outlined in Tables 1 and 2. It should be noted that the computer simulations of the antennas agree with the actual structures in the following critical areas: weight distribution, total weight, projected (normal to blast direction) cross-sectional area distribution, and total projected cross-sectional area.

Prior to the blast trial, a static load was applied near the top of each antenna using an anchored steel cable at a pull angle of 30° to the horizontal. The load was subsequently released electrically ("Twang Test", see [12,13]) and the strain data for free vibration were recorded. A Fourier analysis was performed for the experimental strain data to determine the natural frequencies of the antennas. A comparison of theoretical (numerical simulation) and experimental ("Twang Test") natural frequencies for the two antennas is presented in Table 3. It is apparent from this comparison that the predicted frequencies are in good agreement with the values obtained experimentally.

A comparison of theoretical (numerical simulation) and experimental (blast trial) bending strain histories is presented in Figures 3 and 4. The theoretical predictions were generated using a Friedlander overpressure wave (Figure 5) which corresponds to the Defense Nuclear Agency (DNA) pre-trial predictions for peak overpressure (7.0 psi), positive duration (242 msec) and positive phase impulse (600 psi-msec). The free-field overpressures at the base of the antennas were measured with four strain-type pressure transducers.

The Friedlander overpressure wave which corresponds to the average experimental peak overpressure (6.6 psi), positive duration (251 msec), and positive phase impulse (705 psi-msec) is shown in Figure 5 for comparison purposes.

It is apparent from Figure 3 that the Whip Antenna bending strain predictions are generally much larger than the corresponding experimental strains (the differences are within experimental error bands, considering pressure transducer errors, drag coefficient errors, material uncertainties, etc.). The average ratio of peak theoretical to experimental bending strain from all five Whip Antenna gauge pairs is 1.65 (Table 4). This may be compared to the results in Figure 4 for the Polemast Antenna in which there is excellent agreement between the theoretical and experimental strains. The average ratio of peak theoretical to experimental bending strain from all five Polemast Antenna gauge pairs is 1.19 (Table 4). A considerably more detailed evaluation of the above two antenna experiments may be found in references [12] and [13].

6. Conclusions

A numerical procedure was developed to predict the elastic response of variable cross-section cantilever beams when subjected to a transient air blast load. The numerical model was used to predict the time response of a 35 ft fibreglass Whip Antenna and a 23 ft UHF Polemast Antenna when subjected to a nominal 7.0 psi peak overpressure blast loading. The computed natural frequencies and transient strains were in reasonable agreement with values obtained experimentally in Event Dice Throw. A more detailed evaluation of the above two antenna experiments may be found in references [12] and [13].

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x (in)	ID[1] (in)	OD[1] (in)	OD[2] (in)
0.00[4]	4.400	6.500[3]	6.500[3]
41.85	4.288	5.942	5.942
83.70	4.002	4.878	4.878
125.55	3.665	4.459	4.459
167.40	3.403	4.154	4.154
209.25	3.056	3.935	3.935
251.10	2.707	3.241	3.241
292.95	2.298	2.638	2.638
334.80	1.957	2.257	2.257
376.65	1.695	2.030	2.030
418.50[4]	1.500	1.900[3]	1.900[3]

[1] This profile establishes the mass distribution.

[2] This profile establishes the projected (normal to blast direction) cross-sectional area distribution.

[3] Extrapolated value based on data supplied by the manufacturer.

[4] Boundary conditions: clamped at x=0 in, free at x=418.50 in.

E = 3.9×10^6 psi
ρ = 0.002298 slugs/in^3
Δx = 41.85 in
N = 10
L = 418.5 in (34.88 ft)
P_A = 12.58 psi
T_A = 54.0°F
p_o = 7.0 psi
t_d = 242 msec
I_d = 600 psi-msec (κ=1.137, computed)
Δt = 1.00 msec

The time response is formed using only the lowest 3 natural frequencies and corresponding normal modes.

TABLE 1: Numerical model simulation of a 35 ft fibreglass Whip Antenna. All symbols are defined in the text. The antenna outside diameter (OD) and inside diameter (ID) data were interpolated from data supplied by the manufacturer. The air blast predictions were provided by the United States Defense Nuclear Agency.

x (in)	ID[1] (in)	OD[1] (in)	OD[2] (in)
0[3]	8.978	9.500	9.500
36[3]	8.978	9.500	9.500
72	8.978	9.500	9.500
108	7.950	9.500	17.220
144	8.978	9.500	9.500
180	8.600	9.500	13.360
216	8.600	9.500	13.360
252	8.290	9.500	10.080
288[3]	8.290	9.500	10.080

[1] This profile is calculated to establish the correct mass distribution, assuming a fixed OD equal to that of the seamless extruded aluminum tubing which constitutes the primary structural portion of the antenna.

[2] This profile is calculated to establish the correct projected (normal to blast direction) cross-sectional area distribution.

[3] Boundary conditions: pin at x=0 in, pin at x=36 in, free at x=288 in.

E = 10×10^6 psi
ρ = 0.003044 slugs/in^3
Δx = 36.0 in
N = 8
L = 288.0 in (24.0 ft)
P_A = 12.58 psi
T_A = 54.0°F
P_o = 7.0 psi
t_d = 242 msec
I_d = 600 psi-msec (κ=1.137, computed)
Δt = 1.00 msec

The time response is formed using only the lowest 3 natural frequencies and corresponding normal modes.

TABLE 2: Numerical model simulation of a UHF Polemast Antenna. All symbols are defined in the text. The structural data were obtained from drawings supplied by DMCS-6 (modifications as noted in [13]). The air blast predictions were provided by the United States Defense Nuclear Agency.

Mode	Whip Frequencies (cps)		Polemast Frequencies (cps)	
	Theoretical	Experimental	Theoretical	Experimental
1	1.47	1.27	4.62	4.00
2	4.09	4.20	25.5	24.1[1]
3	9.55	9.50	72.4	—

[1] This value represents an average of indistinct frequencies which appear in a band over the range 19.7 to 32.1 cps.

TABLE 3: A comparison of theoretical (numerical simulation) and experimental (Twang Test) natural frequencies for a 35 ft fibreglass Whip Antenna and a 23 ft UHF Polemast Antenna.

Gauge	Peak Bending Strains (μin/in)					
	Whip Antenna			Polemast Antenna		
	Theoretical	Experimental	Theo./Exp.	Theoretical	Experimental	Theo./Exp.
1	2050	2009	1.02	208	132	1.58
2	3443	2381	1.45	1248	973	1.28
3	3112	1335	2.33	2414	2010	1.20
4	3578	2376	1.51	2008	1917	1.05
5	7171	3713	1.93	774	927	0.83
			Avg. 1.65			Avg. 1.19

TABLE 4: Comparison of the peak theoretical and experimental bending strains (first cycle only) for the Whip and Polemast Antennas.

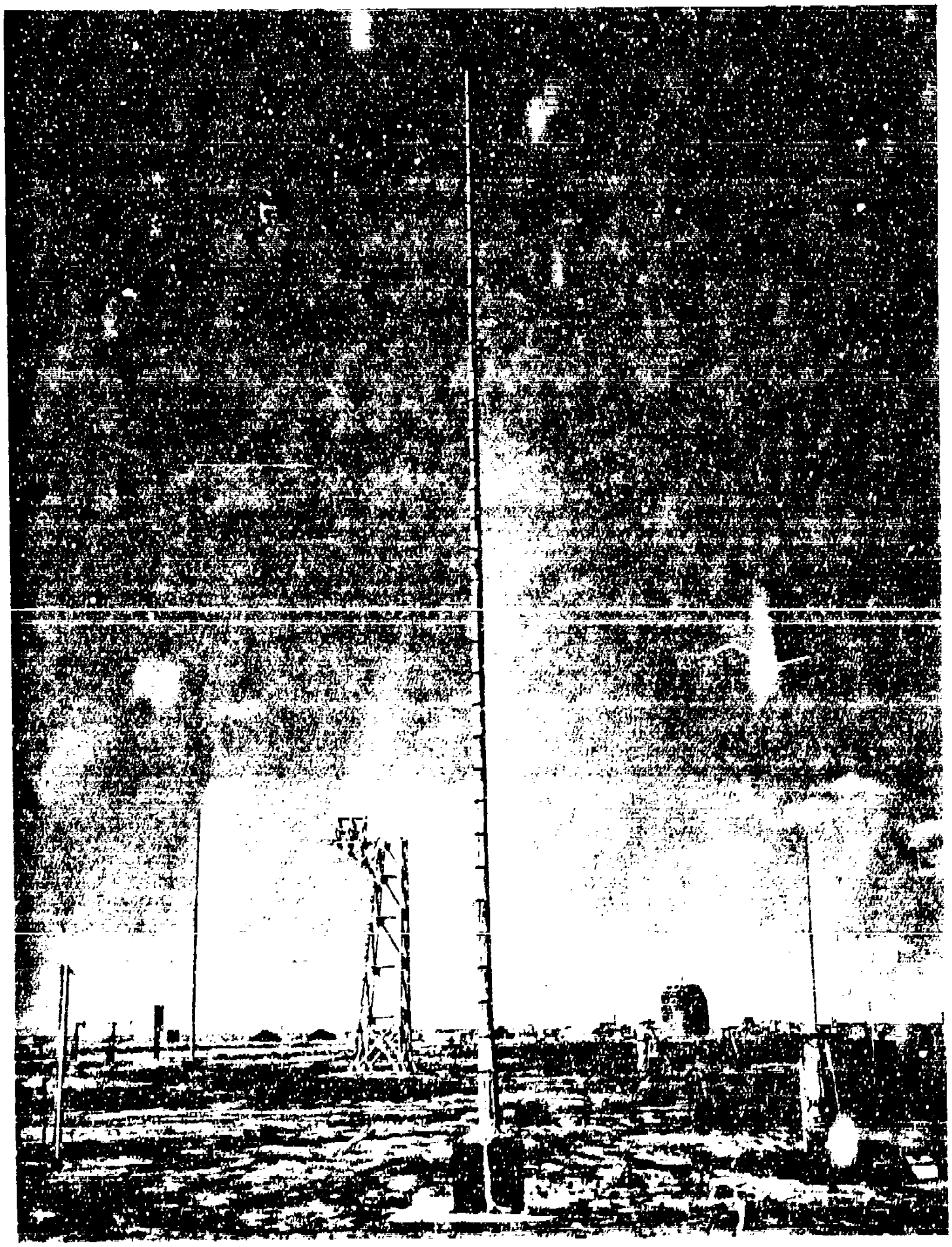

FIGURE 1: Photograph of the 35 ft fibreglass Whip Antenna installed at the nominal 7.0 psi location in Event Dice Throw.

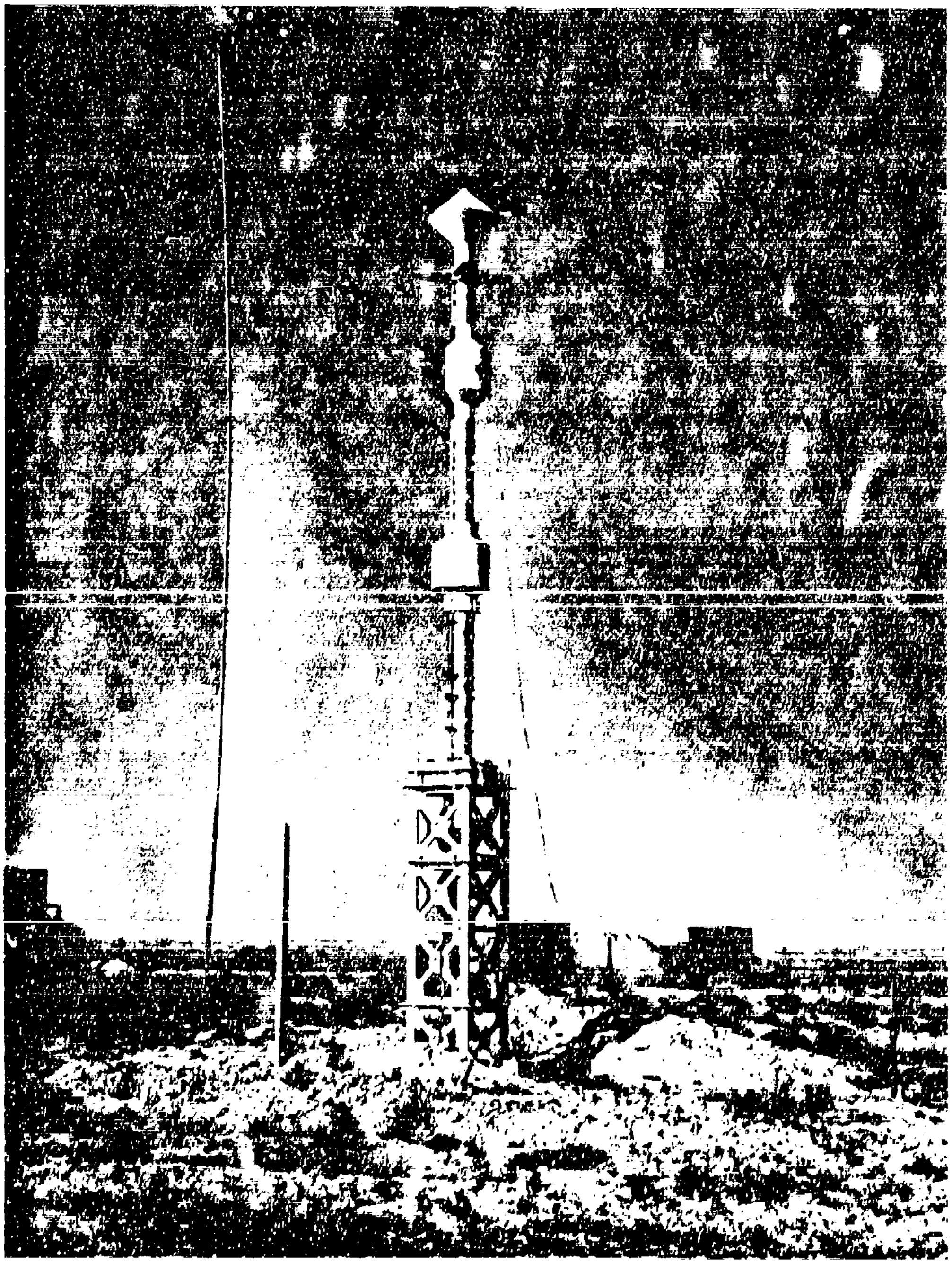

FIGURE 2: Photograph of the 23 ft UHF Polemast antenna installed at the nominal 7.0 psi location in Event Dice throw.

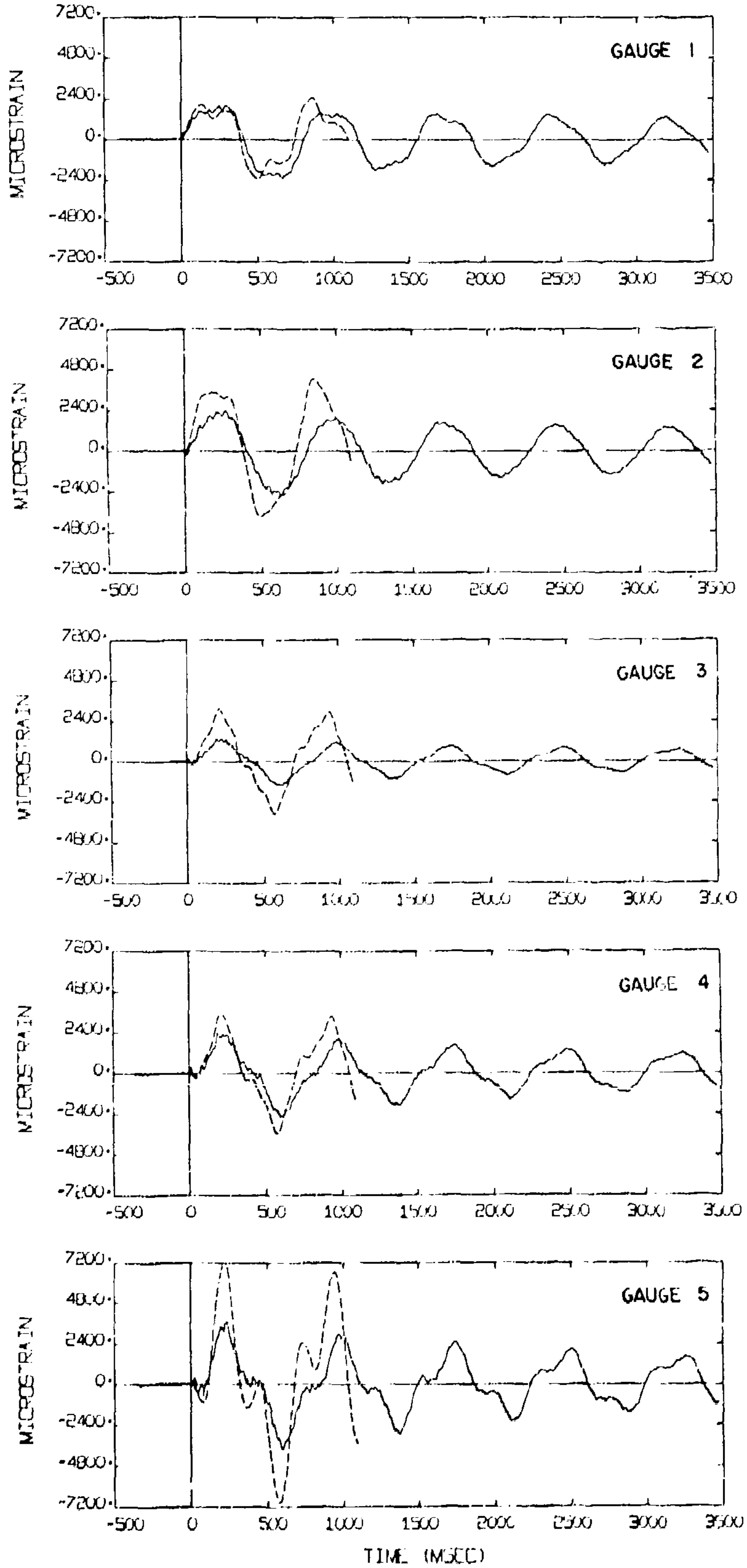

FIGURE 3: Comparison of the Whip Antenna bending strain predictions (dashed lines) against the measured strains (solid lines). The five gauge pairs are respectively located at distances 3.5, 10.5, 17.0, 18.4 and 24.0 ft from the base of the antenna.

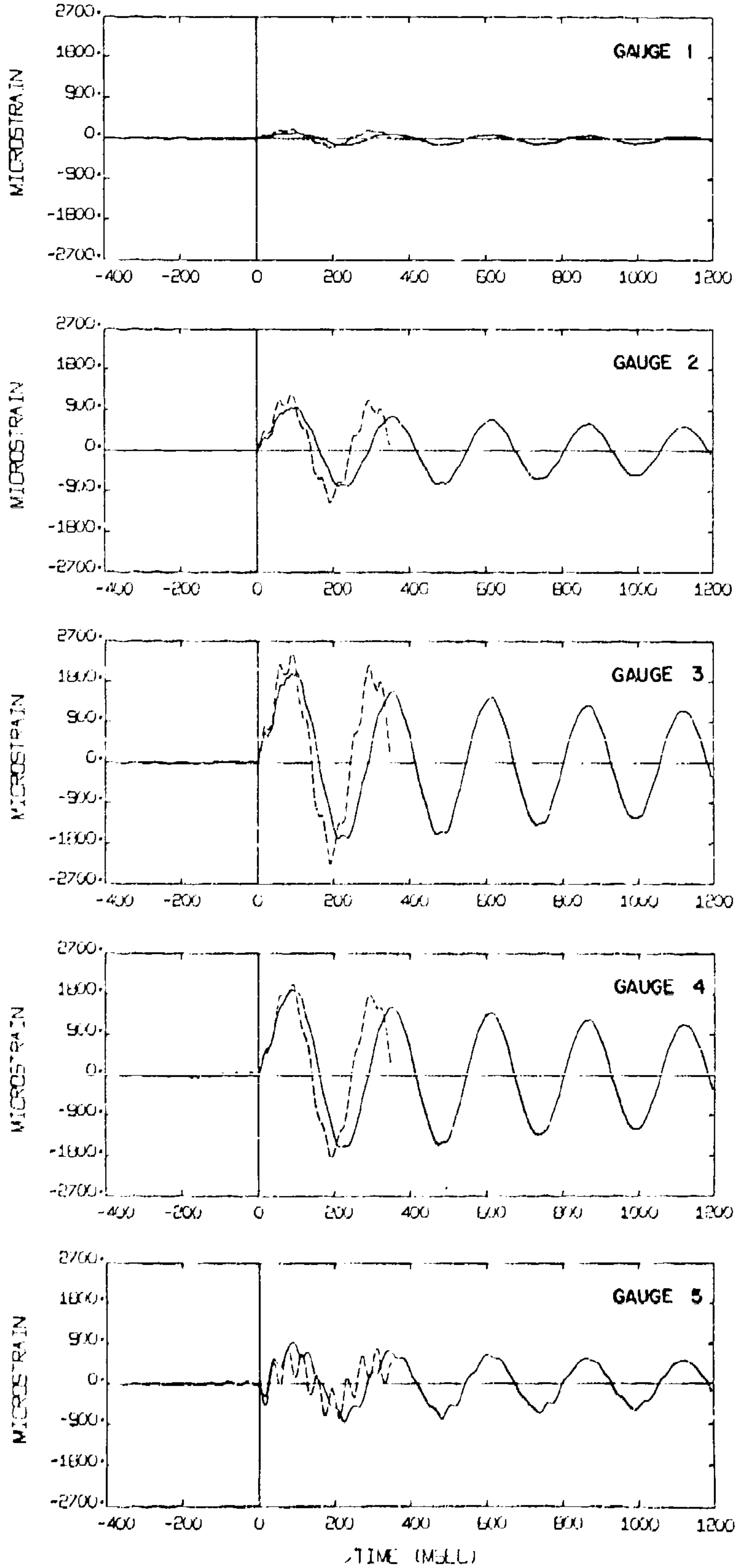

FIGURE 4: Comparison of the Polemast Antenna bending strain predictions (dashed lines) against the measured strains (solid lines). The five gauge pairs are respectively located at distances 0.25, 1.5, 3.25, 4.5 and 12.0 ft from the base of the antenna.

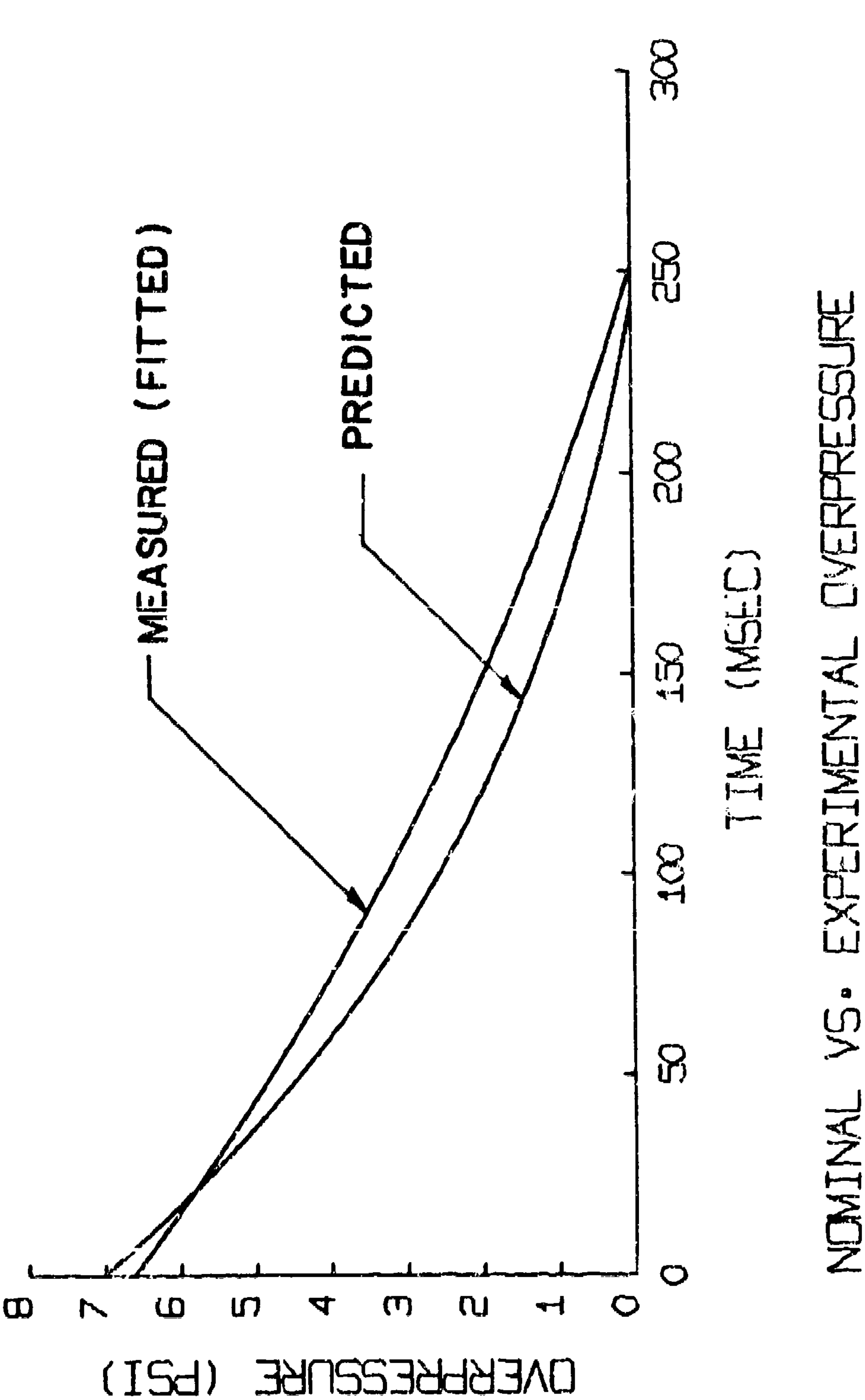

FIGURE 5: Comparison of the Friedlander wave which corresponds to the pre-trial DNA overpressure predictions against the Friedlander wave which corresponds to average experimental overpressure measurements.

APPENDIX A

COMPUTER PROGRAM FOR THE NUMERICAL SIMULATION MODEL

Based on the numerical procedures described in Sections 2, 3 and 4, a computer program was prepared to determine the natural frequencies, normal modes and air blast transient response of a tapered cantilever beam. A listing of the computer program is presented in this appendix, together with a description of the initialization procedures.

(a) Free Vibration

The computer program for free vibration consists of a main program and two subroutines, referred to as "ESOR" and "ANORM". The major portion of the computation for the natural frequencies and normal modes is carried out in subroutine ESOR. On completion of the calculation, the program displays the relevant frequencies and modes, and stores the information in a disk file for future use in the forced vibration calculations. The card input data for the free vibration calculation is outlined below, and a listing of the computer program is provided in Figure A1.

Program Symbol	Text Symbol	Units	Field Type	Description
CARD 1: Iteration control parameters				
N	N	--	I5	number of elements in the beam.
M	-	--	I5	number of frequencies/modes to be calculated, ascending order (3 is satisfactory).
FSTOP	-	--	F10.5	SOR and Rayleigh quotient convergence criteria (.001 is satisfactory.
NSTOP	-	--	I5	iteration loop termination parameter (150 is satisfactory).
IPRNT	-	--	I5	printout option (1=full, 0=partial).
ALEN	L	--	F10.5	length of the beam.
NN	-	--	I5	number of data cards (see Card 2) used to describe the mass distribution along the antenna.
EMOD	E	psi	F10.5	Young's modulus.
RHO	ρ	slugs/in^3	F10.5	density.
ANU	-	--	F10.5	SOR acceleration factor (1.0 is satisfactory).
IBC	I	--	I5	boundary condition code: 0=clamped-free, 1 or larger = pin-pin-free with pin at $x=I\Delta x$.
CARD 2: Description of the mass distribution along the beam; NN cards of this type.				
W1,XX	x	ft	F10.2	position from base of the beam.
W3,DOU	OD	in	F10.2	outside diameter.
W4,DIN	ID	in	F10.2	inside diameter.

FREE VIBRATION

```
C
C     WHIP ANTENNA OR UHF/VHF OR OTHER SINGLE ROD MAST OF VARIABLE CROSS
C     SECTIONAL AREA
C
      DEFINE FILE 1(559,2,U,IB)
      DIMENSION A(40,10),B(10),DIN(44),DOU(44),W1(44),W2(44),W3(44),W4(4
     14)
      DIMENSION AREA(44),AMR(10)
C     ALSO NOTE THAT THE DIMENSIONS FOR A AND B HAVE BEEN SELECTED TO
C     PERMIT VALUES OF N .LE. 40 AND VALUES OF M .LE. 10.   LARGER
C     DIMENSIONS MAY BE SELECTED AT ANY TIME IF REQUIRED
      COMMON DEL,EMOD,RHO,DIN,DOU,W1,W2,W3,W4,AREA,AMR
C     ASSIGN IO DEVICE NUMBERS
      IN=2
      IO=3
C     ASSIGN NN AND MM TO EQUAL THE DIMENSIONS OF A
      NN=40
      MM=10
      NN4=NN+4
      READ (IN,100) N,M,FSTOP,NSTOP,IPRNT,ALEN,NN,EMOD,RHO,ANU,IBC
  100 FORMAT (2I5,F10.5,2I5,F10.5,I5,3F10.5,I5)
      DEL=ALEN/N
      IF (IPRNT) 2,2,1
    1 WRITE (IO,102) N,M,FSTOP,NSTOP
  102 FORMAT ('1'//10X,'NATURAL FREQUENCIES AND NORMAL MODES OF A BEAM W
     1ITH VARIABLE CROSS SECTIONAL AREA'//10X,'BEAM IS DIVIDED INTO N EL
     1EMENTS, N = ',I42/ 10X,'NUMBER OF MODES AND FREQUENCIES TO BE CALC
     1ULATED, IN ASCENDING ORDER, M = ',I5/ 10X,'SOR AND RAYLEIGH QUOTI
     1ENT CONVERGENCE CRITERIA, FSTOP = ',F23.7/  10X,'ITERATION LOOP TE
     1RMINATION PARAMETER, NSTOP = ',I33)
      WRITE (IO,109) ALEN,DEL,EMOD,RHO,ANU,NN,IBC
  109 FORMAT (' ', 9X,'LENGTH OF BEAM (IN), ALEN = ',F51.3/  10X,'BEAM I
     1S DIVIDED INTO ELEMENTS OF LENGTH (IN), DEL = ',F27.3/10X,'YOUNGS
     1MODULUS (PSI), EMOD = ',F50.0/  10X,'BEAM DENSITY (SLUGS/IN**3), R
     1HO = ',F45.9/  10X,'SOR ACCELERATION FACTOR, ANU = ',F48.5/  10X,'
     1NUMBER OF SUPPLIED BEAM COORDINATE POINTS, NN = ',I31/10X,'BOUNDAR
     1Y CONDITION CODE, IBC = ',I48/)
      WRITE (IO,110)
  110 FORMAT (' ', 9X,'BEAM PHYSICAL SPECIFICATIONS'//10X,'X LOCATION (F
     1T)',5X,'X LOCATION (IN)',5X,'INNER DIA (IN)',6X,'OUTER DIA (IN)'/)
    2 DO 3 I=1,NN
      READ (IN,101)  W1(I),W3(I),W4(I)
  101 FORMAT (3F10.2)
      W2(I)=W1(I)*12.
C     W1,W2 DATA VECTORS CONTAIN X LOCATION IN FT AND IN RESPECTIVELY
C     W3,W4 DATA VECTORS CONTAIN THE OUTER AND INNER BEAM DIAMETERS IN
C     INCHES
      IF (IPRNT) 3,3,4
    4 WRITE (IO,104) W1(I),W2(I),W4(I),W3(I)
  104 FORMAT (' ',     4F20.3)
    3 CONTINUE
C
C     NOW CREATE THE INNER AND OUTER DIAMETER VECTORS DIN AND DOU AT THE
C     LOCATIONS X = I*DEL, I=0,1,2,...,N.
      DO 15 I=1,NN4
      AREA(I)=0.
```

```
      SUBROUTINE ESOR(N,M,A,B,FSTOP,NSTOP,IPRNT,ANU,IBC)
C
C     FREQUENCIES AND NORMAL MODES (EIGENVALUES AND EIGENVECTORS)
C     SOR, RAYLEIGH QUOTIENT, AND GRAM SCHMIDT METHODS ARE USED
C
C
C     INPUT INFORMATION .....
C     IBC IS THE BOUNDARY CONDITION PARAMETER
C     IBC = 0 ... CLAMPED - FREE (CANTILEVER) BC, AS IN THE WHIP ANTENNA
C     IBC = NO. .GT. 0 ... PIN - PIN - FREE BC WITH THE PINS AT X = 0
C     AND AT X = IBC*DELTAX AND THE FREE END CONDITION AT X = L.
C     N = NUMBER OF ELEMENTS WHICH THE BEAM IS DIVIDED INTO, OR IS THE
C     NUMBER OF UNKNOWN ELEMENTS IN EACH EIGENVECTOR WHEN SIMPLE CONST-
C     RAINTS AT EACH END ARE USED
C     M = NUMBER OF NORMAL MODES AND FREQUENCIES WHICH ARE TO BE
C     DETERMINED
C     A(I,J),I=1,N, IS USED TO STORE THE J TH NORMAL MODE, J=1 TO M
C     B(J) IS USED TO STORE THE J TH EIGENVALUE
C     NOTE THAT THE EIGENVALUES = CIRCULAR FREQUENCIES ** 2
C     ALSO NOTE THAT THE DIMENSIONS FOR A AND B HAVE BEEN SELECTED TO
C     PERMIT VALUES OF N .LE. 40 AND VALUES OF M .LE. 10.   LARGER
C     DIMENSIONS MAY BE SELECTED AT ANY TIME IF REQUIRED
C     FSTOP IS THE ITERATION CONVERGENCE CRITERIA FOR FRACTIONAL DISPLA-
C     CEMENT NORMS IN THE SOR PROCEDURE AND FRACTIONAL LAMBDA NORM IN
C     THE RAYLEIGH QUOTIENT PROCEDURE
C     NSTOP IS THE ITERATION TERMINATION PARAMETER FOR ANY ITERATION
C     LOOP
C     IPRNT IS THE PRINTOUT OPTION, 1 IS FOR FULL PRINT, 0 IS FOR NO
C     PRINT
C     ANU IS THE SOR ACCELERATION FACTOR
C
C
C     ADDITIONAL INPUT INFORMATION IS SUPPLIED VIA THE COMMON BLOCK
C     ALL UNITS ARE LBF,SLUGS,INCHES,SEC UNLESS OTHERWISE SPECIFIED
C
      DIMENSION A(40,10),B(10),DIN(44),DOU(44),CK(44),EK(44),FK(44),YK(4
     14)
      DIMENSION AREA(44),AMR(10)
      COMMON DEL,EMOD,RHO,DIN,DOU,CK,EK,FK,YK,AREA,AMR
C
C     DEL IS THE GRID SPACING (INCHES)
C     EMOD IS YOUNGS MODULUS (PSI)
C     RHO IS THE BEAM DENSITY (SLUGS/IN**3)
C     DIN AND DOU ARE THE INNER AND OUTER DIAMETERS OF THE ROD AT
C     LOCATIONS SMALL I = 0 TO N, FOR WHICH THE INDEX RANGE IS INDX =
C     2 TO N+2
C     NOTE THAT DIN AND DOU ARE DIMENSIONED N+4  TO ALLOW FOR TWO POINTS
C     AT X= 0 AND -DEL AND TWO POINTS AT X = (N+1)*DEL AND (N+2)*DEL
C
C     IO DEVICE NUMBERS
      IN=2
      IO=3
C     ASSORTED CONSTANTS
      N2=N+2
      PI=3.14159
C     ASSIGNING CK AND EK VECTORS
```

```
110 FORMAT (' ', 9X,'BEAM PHYSICAL SPECIFICATIONS'//10X,'X LOCATION (F
   1T)',5X,'X LOCATION (IN)',5X,'INNER DIA (IN)',6X,'OUTER DIA (IN)'/)
  2 DO 3 I=1,NN
    READ (IN,101)  W1(I),W3(I),W4(I)
101 FORMAT (3F10.2)
    W2(I)=W1(I)*12.
C   W1,W2 DATA VECTORS CONTAIN X LOCATION IN FT AND IN RESPECTIVELY
C   W3,W4 DATA VECTORS CONTAIN THE OUTER AND INNER BEAM DIAMETERS IN
C   INCHES
    IF (IPRNT) 3,3,4
  4 WRITE (IO,104) W1(I),W2(I),W4(I),W3(I)
104 FORMAT (' ',    4F20.3)
  3 CONTINUE
C
C   NOW CREATE THE INNER AND OUTER DIAMETER VECTORS DIN AND DOU AT THE
C   LOCATIONS X = I*DEL, I=0,1,2,....,N.
    DO 15 I=1,NN4
    AREA(I)=0.
    DIN(I)=0.
 15 DOU(I)=0.
    XX=-DEL
    N2=N+2
    DO 5 INDX=2,N2
    XX=XX+DEL
    I=INDX-2
    IF (I) 6,7,6
  7 ILOW=1
    GO TO 10
  6 IF (I-N) 8,9,8
  9 ILOW=NN-1
    GO TO 10
  8 IF (XX-W2(ILOW+1)) 10,10,11
 11 ILOW=ILOW+1
    IF (ILOW-NN+1) 12,10,10
 12 GO TO 8
 10 IHIGH=ILOW+1
    DOU(INDX)=W3(ILOW)+(W3(IHIGH)-W3(ILOW))/(W2(IHIGH)-W2(ILOW))*(XX-W
   12(ILOW))
    DIN(INDX)=W4(ILOW)+(W4(IHIGH)-W4(ILOW))/(W2(IHIGH)-W2(ILOW))*(XX-W
   12(ILOW))
  5 CONTINUE
C   ZERO A AND B
    DO 13 J=1,MM
    AMR(J)=0.
    B(J)=0.
    DO 14 I=1,NN
 14 A(I,J)=0.
 13 CONTINUE
C
C   NORMAL MODES AND FREQUENCIES ARE OBTAINED USING SUBROUTINE ESOR
C
    CALL ESOR(N,M,A,B,FSTOP,NSTOP,IPRNT,ANU,IBC)
C
C
C   CREATE THE GENERALIZED MODAL  PSEUDO MASS  VECTOR IN MODE SPACE

    DO 51 IMODE=1,M
    AMR(IMODE)=0.
    DO 52 J=1,N
 52 AMR(IMODE)=AMR(IMODE)+A(J,IMODE)*A(J,IMODE)*AREA(J+2)
    AMR(IMODE)=AMR(IMODE)*RHO
 51 CONTINUE
C   STORE THE NORMAL MODES, EIGENVALUES, AND SELECTED BEAM CONSTANTS
    AN=1.*N+.1
    AM=1.*M+.1
    AIBC=1.*IBC+.1
    WRITE (1'1) AN,AM,ALEN,DEL,EMOD,RHO,DIN,DOU,A,B,AREA,AMR,AIBC
C
C
    CALL EXIT
    END
```

```
    DIMENSION AREA(44),AMR(10)
    COMMON DEL,EMOD,RHO,DIN,DOU,CK,EK,FK,YK,AREA,AMR
C
C   DEL IS THE GRID SPACING (INCHES)
C   EMOD IS YOUNGS MODULUS (PSI)
C   RHO IS THE BEAM DENSITY (SLUGS/IN**3)
C   DIN AND DOU ARE THE INNER AND OUTER DIAMETERS OF THE ROD AT
C   LOCATIONS SMALL I = 0 TO N, FOR WHICH THE INDEX RANGE IS INDX =
C   2 TO N+2
C   NOTE THAT DIN AND DOU ARE DIMENSIONED N+4  TO ALLOW FOR TWO POINTS
C   AT X= 0 AND -DEL AND TWO POINTS AT X = (N+1)*DEL AND (N+2)*DEL
C
C   IO DEVICE NUMBERS
    IN=2
    IO=3
C   ASSORTED CONSTANTS
    N2=N+2
    PI=3.14159
C
C   ASSIGNING CK AND EK VECTORS
C   ASSIGN AREA VECTOR
    XX=-DEL
    AK=0.
    WTT=0.
    DO 1 INDX=2,N2
    AI=PI/64.*(DOU(INDX)**4.-DIN(INDX)**4.)
    EK(INDX)=EMOD*AI
    IF (INDX-2) 2,2,3
  2 ENOT=EK(2)
    IF (IPRNT) 3,3,4
  4 WRITE (IO,100) ENOT
100 FORMAT ('1'//10X,'NORMAL MODES AND FREQUENCIES USING SOR, RAYLEIGH
   1 QUOTIENT, AND GRAM SCHMIDT TECHNIQUES'//10X,'EPSILON NOUGHT AT RO
   1OT OF BEAM (LBF-IN**2), ENOT = ',E14.5/10X,'NOTE THAT EPSI(K) = E*
   1I = EK*ENOT'//10X,   'PHYSICAL DESCRIPTION OF THE BEAM (ALL UNITS
   1LBF,SLUGS,IN,SEC)'/)
    WRITE (IO,110)
110 FORMAT (' ',2X, 'LOCN',6X,'X (IN)',8X,'OUT DIA (IN)',  2X, 'INN DI
   1A (IN)', 2X, 'AREA (IN**2)',2X,'I (IN**4)', 5X, 'WT (LBF)', 7X, '
   1EK',  4X, 'CK (1/SEC**2)'/)
  3 EK(INDX)=EK(INDX)/ENOT
    AKK=AK
    AK=PI*(DOU(INDX)**2.-DIN(INDX)**2.)/4.
    AREA(INDX)=AK
    CK(INDX)=ENOT/(RHO*DEL**4.*AK)
C   IN THE ABOVE EQUATION, DUE TO THE UNITS CHOSEN, CK HAS UNITS LBF/
C   (IN-SLUG).  TEMPORARILY THIS IS CONVERTED INTO FT SO THAT LBF MAY
C   BE REPLACED WITH SLUG-FT/SEC**2.  THEN THE FINAL ANSWER FOR CK
C   WILL HAVE THE UNITS 1/SEC**2
    CK(INDX)=CK(INDX)*12.
    XX=XX+DEL
    I=INDX-2
    WT=.5*(AKK+AK)*DEL*RHO*32.2
    IF (I) 30,31,30
 31 WT=0.
 30 WTT=WTT+WT
    IF (IPRNT) 1,1,6
  6 WRITE (IO,101) I,XX,DOU(INDX),DIN(INDX),AK,AI,WT,EK(INDX),CK(INDX)
101 FORMAT (' ',  2X,I3,F13.3,F16.3,3F14.3,F12.2,F14.6,E14.6)
  1 CONTINUE
    IF (IPRNT) 32,32,33
 33 WRITE (IO,107) WTT
107 FORMAT (' '//10X,'TOTAL WEIGHT OF THE BEAM (LBF), WTT = ',F10.3//)
C   EXTRAPOLATE EK ONE DELTA X
 32 EK(N+3)=2.*EK(N+2)-EK(N+1)
C
C
C
C   SOR/RAYLEIGH QUOTIENT/GRAM SCHMIDT SOLUTION PROCEDURE
C
```

FIGURE A1: Free vibration computer program listing.

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2

FREE VIBRATION (CONT'D)

```
C
C
      DO 7 IMODE=1,M
C
C
C     ESTABLISH STARTING ESTIMATES FOR EIGENVALUES AND EIGENVECTORS
      DO 20 INDX=3,N2
   20 YK(INDX)=1.0
C     ONE BC AT ROOT OF THE BEAM
      YK(2)=0.
      IF (IBC) 51,51,52
   52 YK(IBC+2)=0.
   51 IJKLM=1
      OMEG2=0.
      IF (IMODE-1) 21,21,22
   22 OMEG2=B(IMODE-1)
   21 ITLP2=0
      ITLP1=0
C
C
C     ITERATION LOOP FOR EACH MODE
      DO 19 IMD=1,NSTOP
      ITLP2=ITLP2+1
C
C
C     SUCCESSIVE OVER RELAXATION (SOR) LOOPS
      DO 10  ILOOP=1,NSTOP
      ITLP1=ITLP1+1
C
C
C     EXTEND Y
C
      YK(1)=YK(3)
      YK(2)=0.
      FK(2)=YK(2)
      IF (IBC) 53,53,54
   54 YK(IBC+2)=0.
      YK(1)=-YK(3)
   53 IJKLM=1
      YK(N+3)=2.*YK(N+2)-YK(N+1)
      YK(N+4)=4.*YK(N+2)-4.*YK(N+1)+YK(N)
C     DO A SINGLE SOR SWEEP
      DNORM=0.
      DO 8 INDX=3,N2
      GK=EK(INDX+1)*YK(INDX+2)-2.*(EK(INDX+1)+EK(INDX))*YK(INDX+1)-2.*(E
     1K(INDX)+EK(INDX-1))*YK(INDX-1)+EK(INDX-1)*YK(INDX-2)
      DK=(EK(INDX+1)+4.*EK(INDX)+EK(INDX-1))*CK(INDX)
      FK(INDX)=CK(INDX)*GK+DK*YK(INDX)
      CON1=ANU/(DK-OMEG2)*(OMEG2*YK(INDX)-FK(INDX))
      IF (INDX-2-IBC) 55,56,55
   56 CON1=0.
   55 IJKLM=1
      DNORM=DNORM+ABS(CON1)
C     FK WILL BE USED TO STORE OLD YK FOR USE BELOW
      FK(INDX)=YK(INDX)
      YK(INDX)=YK(INDX)+CON1
    8 CONTINUE
      DNORM=DNORM/(N+1)
C
C     RAYLEIGH QUOTIENT CALCULATION FOR A BETTER EIGENVALUE ESTIMATE
      SUM1=0.
      SUM2=0.
      DO 37 INDX=3,N2
C     NOTE THAT FK IS RECALCULATED HERE SINCE SMALL CHANGES IN Y HAVE
C     OCCURRED SINCE FK WAS CALCULATED EARLIER
      GK=EK(INDX+1)*YK(INDX+2)-2.*(EK(INDX+1)+EK(INDX))*YK(INDX+1)-2.*(E
     1K(INDX)+EK(INDX-1))*YK(INDX-1)+EK(INDX-1)*YK(INDX-2)
      DK=(EK(INDX+1)+4.*EK(INDX)+EK(INDX-1))*CK(INDX)
      FK(INDX)=CK(INDX)*GK+DK*YK(INDX)
      SUM1=SUM1+YK(INDX)*FK(INDX)
      SUM2=SUM2+YK(INDX)*YK(INDX)
   37 CONTINUE
      SUM1=SUM1/SUM2
      FLN=ABS(SUM1-OMEG2)/SUM1
      OMEG2=SUM1
      B(IMODE)=OMEG2
C     ITERATION LOOP CUMULATIVE TERMINATION CRITERIA
C
      IF (ITLP1- 999) 43,43,18
   43 IJKLM=1
      IF (FLN) 48,49,49
   48 FLN=999.9999999
   49 IF (FDN-FSTOP) 47,19,19
   47 IF (FLN-FSTOP) 18,19,19
   19 CONTINUE
   18 IJKLM=1
C
C
C     PRINTOUT OF THE FINAL EIGENVALUES AND EIGENVECTORS
      IF (IPRNT) 7,7,38
   38 OMEGA=SQRT(OMEG2)
      FREQ=OMEGA/2./PI
      WRITE (IO,103) IMODE,FSTOP,NSTOP,ANU,FIN
  103 FORMAT ('1'//10X,'MODE  ', I5//10X,'SOR / RAYLEIGH QUOTIENT / GRAM
     1 SCHMIDT ITERATION CUMULATIVE PARAMETERS'/10X,'FDN AND FLN CONVERG
     1ENCE CRITERIA, FSTOP = ',F24.7/10X,'ITERATION LOOP UPPER LIMIT, NS
     1TOP = ',I30/  10X,'SUCCESSIVE OVER RELAXATION ACCELERATION FACTOR,
     1 ANU = ',F12.5/   10X,'FINAL FRACTIONAL INTERMEDIATE NORM, FIN =
     1      ',F19.8)
      WRITE (IO,117) FDN
  117 FORMAT (' ',9X,        'FINAL SOR  FRACTIONAL DISPLACEMENT NORM, FD
     1N = ',F19.8)
      WRITE (IO,104) FLN,ITLP2,ITLP1,OMEGA,FREQ
  104 FORMAT (' ', 9X,'FINAL FRACTIONAL LAMBDA (EIGENVALUE) NORM, FLN =
     1', F17.8/    10X,'RAYLEIGH QUOTIENT ITERATION LOOP COUNT, ITLP2 = '
     1,I18/    10X,'SOR ITERATION LOOP CUMULATIVE COUNT, ITLP1 = ',I21/
     110X,'CIRCULAR FREQUENCY (RAD/SEC), OMEGA = ',F28.5//  10X,'FREQUEN
     1CY (CYCLES/SEC), FREQ = ',F12.5//10X,    'NORMALIZED MODE SHAPE'//1
     10X,  'LOCATION',10X,'MODE SHAPE'//)
      DO 25 INDX=2,N2
      I=INDX-2
      CON1=0.
      IF (I) 26,26,27
   27 CON1=A(I,IMODE)
   26 WRITE (IO,109) I,CON1
  109 FORMAT (' ',9X,I8,F23.7)
```

```
C     SUCCESSIVE OVER RELAXATION (SOR) LOOPS
      DO 10  ILOOP=1,NSTOP
      ITLP1=ITLP1+1
C
C
C     EXTEND Y
C
      YK(1)=YK(3)
      YK(2)=0.
      FK(2)=YK(2)
      IF (IBC) 53,53,54
   54 YK(IBC+2)=0.
      YK(1)=-YK(3)
   53 IJKLM=1
      YK(N+3)=2.*YK(N+2)-YK(N+1)
      YK(N+4)=4.*YK(N+2)-4.*YK(N+1)+YK(N)
C     DO A SINGLE SOR SWEEP
      DNORM=0.
      DO 8 INDX=3,N2
      GK=EK(INDX+1)*YK(INDX+2)-2.*(EK(INDX+1)+EK(INDX))*YK(INDX+1)-2.*(E
     1K(INDX)+EK(INDX-1))*YK(INDX-1)+EK(INDX-1)*YK(INDX-2)
      DK=(EK(INDX+1)+4.*EK(INDX)+EK(INDX-1))*CK(INDX)
      FK(INDX)=CK(INDX)*GK+DK*YK(INDX)
      CON1=ANU/(DK-OMEG2)*(OMEG2*YK(INDX)-FK(INDX))
      IF (INDX-2-IBC) 55,56,55
   56 CON1=0.
   55 IJKLM=1
      DNORM=DNORM+ABS(CON1)
C     FK WILL BE USED TO STORE OLD YK FOR USE BELOW
      FK(INDX)=YK(INDX)
      YK(INDX)=YK(INDX)+CON1
    8 CONTINUE
      DNORM=DNORM/(N+1)
      CALL ANORM(YK,2,N2,YNORM,2)
      FIN=DNORM/YNORM
C     STORE YK(INDX) IN A(I,IMODE)
      DO 11 I=1,N
   11 A(I,IMODE)=YK(I+2)
C
C
C     GRAM-SCHMIDT ORTHOGONALIZATION
C     SEE ANTENNA NOTES REGARDING THE MATRIX WHICH
C     THE NATURAL MODES ARE ORTHOGONALIZED WHITH RESPECT TO A WEIGHTING
C     FUNCTION
      IMM=IMODE-1
      IF (IMM) 12,12,13
   13 DO 14 IORTO=1,IMM
      SUM1=0.
      SUM2=0.
      DO 15 I=1,N
      SUM1=SUM1+A(I,IMODE)*A(I,IORTO)*AREA(I+2)
      SUM2=SUM2+A(I,IORTO)*A(I,IORTO)*AREA(I+2)
   15 CONTINUE
      DO 16 I=1,N
   16 YK(I+2)=YK(I+2)-A(I,IORTO)*SUM1/SUM2
   14 CONTINUE
      CALL ANORM(YK,2,N2,YNORM,2)
   12 IJKLM=1
      DO 17 I=1,N
      FK(I+2)=FK(I+2)-YK(I+2)
   17 A(I,IMODE)=YK(I+2)
      FK(2)=0.
      CALL ANORM(FK,2,N2,DNORM,1)
      FDN=DNORM/YNORM
      IF (FDN-FSTOP) 9,10,10
   10 CONTINUE
    9 IJKLM=1
   49 IF (FDN-FSTOP) 47,19,19
   47 IF (FLN-FSTOP) 18,19,19
   19 CONTINUE
   18 IJKLM=1
C
C
C     PRINTOUT OF THE FINAL EIGENVALUES AND EIGENVECTORS
      IF (IPRNT) 7,7,38
   38 OMEGA=SQRT(OMEG2)
      FREQ=OMEGA/2./PI
      WRITE (IO,103) IMODE,FSTOP,NSTOP,ANU,FIN
  103 FORMAT ('1'//10X,'MODE  ', I5//10X,'SOR / RAYLEIGH QUOTIENT / GRAM
     1 SCHMIDT ITERATION CUMULATIVE PARAMETERS'/10X,'FDN AND FLN CONVERG
     1ENCE CRITERIA, FSTOP = ',F24.7/10X,'ITERATION LOOP UPPER LIMIT, NS
     1TOP = ',I30/  10X,'SUCCESSIVE OVER RELAXATION ACCELERATION FACTOR,
     1 ANU = ',F12.5/   10X,'FINAL FRACTIONAL INTERMEDIATE NORM, FIN =
     1     ',F19.8)
      WRITE (IO,117) FDN
  117 FORMAT (' ',9X,       'FINAL SOR  FRACTIONAL DISPLACEMENT NORM, FD
     1N = ',F19.8)
      WRITE (IO,104) FLN,ITLP2,ITLP1,OMEGA,FREQ
  104 FORMAT (' ', 9X,'FINAL FRACTIONAL LAMBDA (EIGENVALUE) NORM, FLN =
     1', F17.8/   10X,'RAYLEIGH QUOTIENT ITERATION LOOP COUNT, ITLP2 = '
     1,I18/    10X,'SOR ITERATION LOOP CUMULATIVE COUNT, ITLP1 = ',I21/
     110X,'CIRCULAR FREQUENCY (RAD/SEC), OMEGA = ',F28.5//  10X,'FREQUEN
     1CY (CYCLES/SEC), FREQ = ',F12.5//10X,    'NORMALIZED MODE SHAPE'//1
     10X,  'LOCATION',10X,'MODE SHAPE'//)
      DO 25 INDX=2,N2
      I=INDX-2
      CON1=0.
      IF (I) 26,26,27
   27 CON1=A(I,IMODE)
   26 WRITE (IO,109) I,CON1
  109 FORMAT (' ',9X,I5,F23.7)
   25 CONTINUE
C     ORTHOGONALITY CHECK
      WRITE (IO,115) IMODE
  115 FORMAT (' '//10X,'ORTHOGONALITY CHECK FOR EIGENVECTOR MODE',I5/)
      DO 41 IORTO=1,IMODE
      SUM1=0.
      DO 42 I=1,N
   42 SUM1=SUM1+A(I,IMODE)*A(I,IORTO)*AREA(I+2)
      SUM1=SUM1*RHO
      WRITE (IO,116) IMODE,IORTO,SUM1
  116 FORMAT (' ',9X,'MODES',I5,'     AND',I5,'   ,  WEIGHTED INNER PRODUC
     1T IS',F12.7)
   41 CONTINUE
    7 CONTINUE
      RETURN
      END


      SUBROUTINE ANORM(Y,M,N,YNORM,ICODE)
C     THIS SUB COMPUTES THE NORM OF Y(I), I = M TO N
C     THE NORM IS RETURNED IN YNORM
C     YNORM = SUM (ABS(Y(I)))/ (NUMBER OF ELEMENTS IN Y)
C     WHEN ICODE IS 1, ONLY THE NORM OF Y IS COMPUTED
C     WHEN ICODE IS 2, YNORM IS COMPUTED AND Y ITSELF IS NORMALIZED
      DIMENSION Y(44)
      YNORM=0.
      DO 1 I=M,N
      CON1=Y(I)
      YNORM=YNORM+ABS(CON1)
    1 CONTINUE
      YNORM=YNORM/(N-M+1)
      IF (ICODE-2) 3,2,3
    2 DO 4 I=M,N
    4 Y(I)=Y(I)/YNORM
    3 RETURN
      END
```

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(b) Forced Vibration

The computer program for forced vibration consists of a main program and three subroutines, referred to as "DRAGC", "YDISP" and "VISCO". The major portion of the computation for the forced vibration response is carried out in the main program. Subroutines DRAGC, YDISP and VISCO respectively deal with computation of the air blast loading on circular cylinders, display of the stresses which correspond to a specified beam displacement profile, and calculation of the kinematic viscosity for air. Input data for this program consists of the natural frequencies and normal modes computed previously (read from a disk file) and data cards identifying the air blast loading and assorted parameters required in the time response calculation. The card input data for the forced vibration calculation is outlined below, and a listing of the computer program is provided in Figure A2.

Program Symbol	Text Symbol	Units	Field Type	Description
Card 1: Data describing the normal mode plots.				
INPLT	-	--	I5	plotting option (0=suppress the plot).
ICODE	-	--	I5	printout option in the plot subroutine.
XLEN	-	in	F10.3	length of the abscissa.
YLEN	-	in	F10.3	length of the ordinate.
YDELL	-	--	F10.3	scale of the ordinate.

Program Symbol	Text Symbol	Units	Field Type	Description
Card 2: Data describing the air blast loading on the antenna.				
PO	p_o	psi	F10.1	peak overpressure.
TPLUS	t_d	sec	F10.1	positive phase duration.
PA	P_A	psi	F10.1	atmospheric pressure.
TA	T_A	°F	F10.1	atmospheric temperature.
D	-	in	F10.1	representative antenna diameter (for display of drag pressures only).
AK	κ	--	F10.1	Friedlander decay constant.
TIME	-	sec	F10.1	time step (for display of drag pressures only; .010 is satisfactory).
LOOPS	-	--	I10	limit on the number of display loops (display purposes only; 999 is satisfactory).
Card 3: Number of cards needed to specify the projected (normal to blast direction) cross-sectional area profile of the antenna.				
NN	-	--	I5	if NN=1, then the OD used in the Free Vibration problem also specifies the projected antenna area profile; if NN>1, then there are NN cards to follow to specify this profile (see Card 4).
Card 4: Description of the projected (normal to blast direction) cross-sectional area profile of the antenna; NN cards of this type (if NN≥1).				
W1,XX	x	ft	F10.2	position from base of the beam.
W3,DIA	OD	in	F10.2	outside diameter.
Card 5: Control parameters for the transient response calculation.				
MODES	-	--	I10	number of normal modes used in constructing the time response (ascending order in frequency).
DELT	Δt	sec	F10.5	time step used in the numerical integration.
TLIM	-	sec	I10	maximum time to which the time response is carried.
MDISP	-	--	F10.5	the number of time steps between beam response displays.

FORCED VIBRATION

```
C
C     RETRIEVE NORMAL MODES, GENERATE STRESSES AND PLOTS, AND GENERATE
C     THE TIME HISTORY RESPONSE
      DEFINE FILE 1(559,2,U,IB)
      DIMENSION A(40,10),B(10),DIN(44),DOU(44),YK(44)
      DIMENSION DIA(44),W1(44),W2(44),W3(44),CBETA(10),SBETA(10),FACTK(1
     10)
      DIMENSION AREA(44),AMR(10)
      COMMON P0,TPLUS,PA,TA,D,AK,GAMMA,R
C     IO DEVICES
      IN=2
      IO=3
      PI=3.14159
C     READ NORMAL MODES AND FREQUENCIES ETC.
      READ (1'1)  AN,AM,ALEN,DEL,EMOD,RHO,DIN,DOU,A,B,AREA,AMR,AIBC
      N=AN
      M=AM
      IBC=AIBC
      WRITE (IO,100) N,M,ALEN,DEL,EMOD,RHO,IBC
  100 FORMAT ('1'/10X,'N,M,ALEN,DEL,EMOD,RHO,IBC',2I5,2F10.4,2F15.5,I5)
      N2=N+2
      NN4=44
      DO 1 INDX=2,N2
      I=INDX-2
      WRITE (IO,101) I,DIN(INDX),DOU(INDX),AREA(INDX)
  101 FORMAT (' ',9X,'I,DIN,DOU,AREA',I5,3F12.6)
    1 CONTINUE
      WRITE (IO,105) (I,AMR(I),I=1,M)
  105 FORMAT ('0',7X,I5,F20.10)
C
C
C     ESTABLISH THE CIRCULAR FREQUENCIES
      DO 22 IMODE=1,M
      CON1=B(IMODE)
      B(IMODE)=SQRT(CON1)
   22 CONTINUE
C
C
C
C     NORMAL MODE/ FREQUENCY DISPLAY
C     INPUT PLOT DATA,  NOTE THAT INPLT = 0 MEANS PLOTTING WILL BE
C     SUPPRESSED
      READ (IN,310) INPLT,ICODE,XLEN,YLEN,YDELL
  310 FORMAT (2I5,3F10.3)
      WRITE (IO,311) INPLT,ICODE,XLEN,YLEN,YDELL
  311 FORMAT ('0'/10X,'PLOT DATA ... INPLT,ICODE,XLEN,YLEN,YDELL ...',2I
     15,3F10.3//)
      DO 2 IMODE=1,M
      DO 3 INDX=3,N2
    3 YK(INDX)=A(INDX-2,IMODE)
      FREQ=B(IMODE)/PI/2.
C
C
      IF (INPLT) 71,72,71
   71 IJKLM=1
      CALL YDISP(IMODE,FREQ,N,YK,DIN,DOU,DEL,EMOD,RHO,ICODE,XLEN,YLEN,YD
     1ELL,IBC)
   72
```

```
   12 GO TO 8
   10 IHIGH=ILOW+1
      DIA(INDX)=W3(ILOW)+(W3(IHIGH)-W3(ILOW))/(W2(IHIGH)-W2(ILOW))*(XX-W
     12(ILOW))
    5 CONTINUE
   19 IJKLM=1
C
C
C     ESTABLISH CONSTANTS FOR THE TIME HISTORY
      READ (IN,500) MODES,DELT,MDISP,TLIM
  500 FORMAT (I10,F10.5,I10,F10.5)
C
C     DISPLAY CONSTANTS, DRAG DIAMETERS, ETC.
      WRITE (IO,600) MODES,DELT,TLIM,MDISP
  600 FORMAT ('1'//10X,'TIME HISTORY RESPONSE OF VARIABLE AREA BEAM'//
     110X,'NUMBER OF NORMAL MODES USED IN CONSTRUCTING THE TIME RESPONSE
     1, MODES = ',I10/  10X,'TIME STEP USED IN THE TIME INTEGRATIONS (SE
     1C), DELT = ',F27.6/   10X,'TRANSIENT RESPONSE WILL BE CARRIED OUT
     1TO THE TIME LIMIT (SEC), TLIM = ',F10.6/   10X,'NUMBER OF TIME STE
     1PS BETWEEN BEAM RESPONSE DISPLAYS, MDISP = ',I20/)
      WRITE (IO,601)
  601 FORMAT (' '/10X,'EFFECTIVE EXTERNAL BEAM DIAMETER PROFILE FOR THE
     1DRAG CALCULATION'//10X,'LOCN',12X,'DIAMETER (IN)',7X  ,'PROJECTED
     1AREA (FT**2) PER LINEAR FOOT OF BEAM'/)
      DO 21 INDX=2,N2
      I=INDX-2
      AAAA=DIA(INDX)/12.
      WRITE (IO,602) I,DIA(INDX),AAAA
  602 FORMAT (' ',I13,F23.4,30X,F10.6)
   21 CONTINUE
C
C
      NTIME=TLIM/DELT+1
      IF (NTIME-1000) 25,25,26
   26 NTIME=1000
   25 IF (MODES-M) 27,27,28
   28 MODES=M
   27 IJKLM=1
      LOOPS=1
      TIMF=-DELT
      DO 33 K=1,MODES
      CBETA(K)=0.
      SBETA(K)=0.
   33 CONTINUE
      IOUT=MDISP
C     INITIALIZE Y
      DO 60 I=1,NN4
   60 YK(I)=0.
C
C
C
C
      DO 29 ILOOP=1,NTIME
      TIME=TIME+DELT
      TREAL=TIME+.5*DELT
      DO 30 I=1,N
      D=DIA(I+2)
      CALL DRAGC(LOOPS,0,TIME,DRAG)
```

```
C
C     NORMAL MODE/ FREQUENCY DISPLAY
C     INPUT PLOT DATA.  NOTE THAT INPLT = 0 MEANS PLOTTING WILL BE
C     SUPPRESSED
      READ (IN,310) INPLT,ICODE,XLEN,YLEN,YDELL
  310 FORMAT (2I5,3F10.3)
      WRITE (IO,311) INPLT,ICODE,XLEN,YLEN,YDELL
  311 FORMAT ('0'/10X,'PLOT DATA ... INPLT,ICODE,XLEN,YLEN,YDELL ...',2I
     15,3F10.3//)
      DO 2 IMODE=1,M
      DO 3 INDX=3,N2
    3 YK(INDX)=A(INDX-2,IMODE)
      FREQ=B(IMODE)/PI/2.
C
C
      IF (INPLT) 71,72,71
   71 IJKLM=1
      CALL YDISP(IMODE,FREQ,N,YK,DIN,DOU,DEL,EMOD,RHO,ICODE,XLEN,YLEN,YD
     1ELL,IRC)
   72 IJKLM=1
C
    2 CONTINUE
C
C
C     BASIC BLAST WAVE PROFILE AND SCAN OF DRAG LOADING
C
C
      READ (IN,300) PO,TPLUS,PA,TA,D,AK,TIME,LOOPS
  300 FORMAT (7F10.1,I10)
      GAMMA=1.4
      R=1715.
      CALL DRAGC(LOOPS,1,TIME,DRAG)
C
C
C     READ IN OR ASSIGN DIAMETERS FOR DRAG LOADING CALCULATION
C
C
      READ (IN,400) NN
  400 FORMAT (I5)
C     IF NN = 1, THEN THE DIAMETERS FOR THE DRAG LOADING CALCULATION ARE
C     THE OUTER DIAMETER VECTOR DIA
C     IF NN .GT. 1, THEN THE DIAMETERS TO BE READ IN BELOW SPECIFY THE
C     FACE-ON DIAMETERS FOR USE IN THE DRAG LOADING CALCULATION
      IF (NN-1) 16,16,17
   16 DO 18 INDX=2,N2
   18 DIA(INDX)=DOU(INDX)
      GO TO 19
   17 DO 20 I=1,NN
      READ (IN,401) W1(I),W3(I)
  401 FORMAT (2F10.2)
      W2(I)=W1(I)*12.
C     W1,W2 DATA VECTORS CONTAIN X LOCATION IN FT AND IN RESPECTIVELY
C     W3    DATA VECTOR CONTAINS THE OUTER            BEAM DIAMETER  IN
C     INCHES
      WRITE (IO,104) W1(I),W2(I),      W3(I)
  104 FORMAT (' ',     3F20.3)
   20 CONTINUE
C
C     NOW CREATE THE           OUTER DIAMETER VECTORS DIA          AT THE
C     LOCATIONS X = I*DEL, I=0,1,2,...,N.
      DO 15 I=1,NN4
   15 DIA(I)=0.
      XX=-DEL
      N2=N+2
      DO 5 INDX=2,N2
      XX=XX+DEL
      I=INDX-2
      IF (I) 6,7,6
    7 ILOW=1
      GO TO 10
    6 IF (I-N) 8,9,8
    9 ILOW=NN-1
      GO TO 10
    8 IF (XX-W2(ILOW+1)) 10,10,11
   11 ILOW=ILOW+1
      IF (ILOW-NN+1) 12,10,10
```

```
      LOOPS=1
      TIME=-DELT
      DO 33 K=1,MODES
      CBETA(K)=0.
      SBETA(K)=0.
   33 CONTINUE
      IOUT=MDISP
C     INITIALIZE Y
      DO 60 I=1,NN4
   60 YK(I)=0.
C
C
C
C
      DO 29 ILOOP=1,NTIME
      TIME=TIME+DELT
      TREAL=TIME+.5*DELT
      DO 30 I=1,N
      D=DIA(I+2)
      CALL DRAGC(LOOPS,0,TIME,DRAG)
      W1(I)=DRAG*D/12.
C     REDUCTION OF LOADING DUE TO REDUCED PROJECTED AREA WHEN THE
C     ANTENNA IS AT A FINITE SLOPE RELATIVE TO THE VERTICAL
      CON1=(YK(I+3)-YK(I+1))*12./2./DEL
      CON1=ABS(CON1)
      CON1=ATAN(CON1)
      W1(I)=W1(I)*COS(CON1)
   30 CONTINUE
      CON1=DELT
      IF (ILOOP-1) 34,34,35
   34 CON1=.5*DELT
   35 DO 31 IMODE=1,MODES
      BETAK=0.
      DO 32 I=1,N
      BETAK=BETAK+A(I,IMODE)*W1(I)
   32 CONTINUE
      ARG=B(IMODE)*TIME
      CBETA(IMODE)=CBETA(IMODE)+CON1*BETAK*COS(ARG)
      SBETA(IMODE)=SBETA(IMODE)+CON1*BETAK*SIN(ARG)
      ARG=B(IMODE)*TREAL
      FACTK(IMODE)=(SIN(ARG)*CBETA(IMODE)-COS(ARG)*SBETA(IMODE))/B(IMODE
     1)/AMR(IMODE)
   31 CONTINUE
C     NB NOTE ... DISPLACEMENTS Y AS A FUNCTION OF TIME ARE GENERATED
C     FOR THE TIME T = TREAL ( NOT FOR THE TIME T = TIME)
      IOUT=IOUT+1
      IF (IOUT-MDISP) 29,39,39
   39 IOUT=0
C     GENERATE Y AT THIS LATER TIME AND DISPLAY PRINTOUT
C     THE CALCULATION TIME IS T = TREAL
      DO 41 I=1,N
      INDX=I+2
      YK(INDX)=0.
      DO 42 IMODE=1,MODES
   42 YK(INDX)=YK(INDX)+A(I,IMODE)*FACTK(IMODE)
C     CONVERT YK FROM INCHES TO FEET
      YK(INDX)=YK(INDX)/12.
   41 CONTINUE
      TMSEC=TREAL*1000.
      IMODE=TMSEC
      WRITE (IO,800) TMSEC,(FACTK(I),I=1,MODES)
  800 FORMAT (' '//10X,'TIME (MSEC)',F9.3  //  10X,'NORMAL MODES RELATIV
     1E CONTRIBUTION (ASCENDING ORDER)',7F10.4/)
      ICODE=3
      CALL YDISP(IMODE,FREQ,N,YK,DIN,DOU,DEL,EMOD,RHO,ICODE,XLEN,YLEN,YD
     1ELL,IRC)
   29 CONTINUE
C
C
C
      CALL EXIT
      END
```

2

FIGURE A2: Forced vibration computer program listing.

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FORCED VIBRATION (CONT'D)

```
      SUBROUTINE DRAGC(ICODE,IPRNT,TIME,DRAG)
C
C     TRANSIENT DRAG ON CIRCULAR CYLINDERS
C
C
C     INPUT INFORMATION .....
C     PO,TPLUS, ETC PARAMETERS IN COMMON MUST BE SUPLIED
C     ICODE IS THE NUMBER OF TIMES THE DRAG CALCULATION IS PERFORMED
C     IPRNT IS THE OUTPUT OPTION, 0 = NO OUTPUT, 1 = FULL PRINTOUT
C     TIME IS THE TIME IN SEC AT WHICH THE DRAG CALCULATION IS TO BE
C     PERFORMED IF ICODE IS 1, OR TIME IS THE TIME INTERVAL IN SECONDS
C     BETWEEN SUCCESIVE DRAG CALCULATIONS IF ICODE .GT. 1
C     DRAG IS THE FINAL CALCULATED LOADING IN POUNDS/FT**2
      COMMON PO,TPLUS,PA,TA,D,AK,GAMMA,R
C
C     ASSIGNMENT OF INPUT/OUTPUT DEVICE NUMBERS (IN,IO)
      IN=2
      IO=3
C
C     LOCATION A REFERS TO AMBIENT CONDITIONS
C     LOCATION B REFERS TO THE REGION IMMEDIATELY BEHIND THE SHOCK FRONT
C     LOCATION C REFERS TO THE IMMEDIATE VICINITY OF THE CYLINDER
C
      DD=D
      D=D/12.
      TA=TA+461.
      CA=SQRT(GAMMA*R*TA)
      PB=PA+PO
      ZETA=PB/PA
C     NOTE THAT SEVERAL OF THE EQUATIONS WHICH FOLLOW ASSUME GAMMA =1.4
      TB=TA*ZETA*(6.+ZETA)/(6.*ZETA+1.)
      CB=CA*SQRT(TB/TA)
      QO=2.5*PO**2./(PO+7.*PA)
      QIO=QO+QO**2./(2.8*PB)
      AMACS =SQRT(1.+6.*PO/(7.*PA))
      UB=5.*PO*CA/(7.*PA*AMACS )
      US=AMACS *CA
      TT=D/US
      AMACB =UB/CB
      ANUB=VISCO (TB,PB,PA,PB)
      REB=UB*D/ANUB
      TDIFR=5.*TT
      IF (IPRNT)2,2,1
    1 WRITE (IO,101) PA,TA,DD,PO,TPLUS,AK
  101 FORMAT ('1'//10X,'TRANSIENT DRAG ON CYLINDERS DUE TO AIR BLAST'//
     110X,'AMBIENT PRESSURE (PSI), PA = ',F16.2/ 10X,'AMBIENT TEMPERATUR
     1E (DEG R), TA = ',F11.2/  10X,'CYLINDER DIAMETER (IN), D = ',F17.3
     1/ 10X,'PEAK OVERPRESSURE (PSI), PO = ',F15.3/ 10X,'POSITIVE DURATI
     1ON (SEC), TPLUS = ',F12.5/ 10X,'DECAY CONSTANT, AK = ',F24.2)
      WRITE (IO,104) CA,US,TT,TDIFR
  104 FORMAT (' '/10X,
     1        'AMBIENT SPEED OF SOUND (FPS), CA = ',F10.2/ 10X,'SPEED OF
     1THE SHOCK FRONT (FPS), US = ',F8.2/  10X,'TIME OF ENGULFMENT (SEC)
     1, TT = ',F14.5/ 10X,'LENGTH OF TIME FOR WHICH THE DIFRACTION PRESS
     [illegible]
```

```
      SUBROUTINE YDISP(IMODE,FREQ,N,YK,DIN,DOU,DEL,EMOD,RHO,ICODE,XLEN,Y
     1LEN,YDELL,IBC)
C
C     NORMAL MODE OR GENERAL CURVE DISPLAY
C
C     IBC IS THE BOUNDARY CONDITION PARAMETER
C     IBC = 0 ... CLAMPED - FREE (CANTILEVER) BC, AS IN THE WHIP ANTENNA
C     IBC = NO. .GT. 0 ... PIN - PIN - FREE BC WITH THE PINS AT X = 0
C     AND AT X = IBC*DELTAX AND THE FREE END CONDITION AT X = L.
C     IMODE IS EITHER THE NORMAL MODE NUMBER OR THE CURVE IDENTIFICA-
C     TION NUMBER
C     ICODE = 1, PRINTOUT AND STRESSES ONLY
C     ICODE = 2, ABOVE PRINTOUTS PLUS PLOT
C     ICODE = 3, ABREVIATED PRINTOUT ONLY
C     DISPLACEMENTS YK ARE IN FEET
C     ALL OTHER DIMENSIONS ARE LBF,SLUGS,IN,SEC
C     XLEN AND YLEN ARE THE PLOT AXES LENGTHS IN INCHES
C     YDELL IS THE DESIRED VARIABLE RANGE BETWEEN UNITS ON THE Y AXIS
C     IF YDELL .LT. 0, THEN A VALUE IS AUTOMATICALLY SELECTED WITHIN
C     THE PROGRAM TO SCALE  DATA TO FIT ON THE PLOT
      DIMENSION YK(44),DIN(44),DOU(44)
C     IO DEVICES
      IN=2
      IO=3
      N2=N+2
      PI=3.14159
C     BC'S ON YK
      YK(2)=0.
      YK(1)=YK(3)
      IF (IBC) 51,51,52
   52 YK(1)=-YK(3)
   51 IJKLM=1
      YK(N+3)=2.*YK(N+2)-YK(N+1)
      YK(N+4)=4.*YK(N+2)-4.*YK(N+1)+YK(N)
      IF (ICODE-2) 2,2,1
    2 WRITE (IO,100) IMODE,FREQ
  100 FORMAT ('1'//10X,'MODE/CURVE NUMBER ',I5
     1                             //10X,'FREQUENCY (CYCLES/SEC)',F12.5//1
     10X,'MOMENTS ARE COMPUTED AT THE GRID POINTS USING 3-POINT OPERATOR
     1S'/10X,'SHEAR FORCES ARE COMPUTED .5*DEL BEHIND THE INDICATED GRID
     1 POINT'//5X,'LOCN',6X,'Y SHAPE (FT)',3X,'SHEAR (LBF)',4X,'MOMENT (
     1IN-LBF)',5X,'SHEAR (PSI)',4X,'FLEXURE (PSI)'/)
      GO TO 32
    1 WRITE (IO,109) IMODE
  109 FORMAT (' '/10X,'TIME (MSEC)',I7//
     1              5X,'LOCN',6X,'Y SHAPE (FT)',3X,'SHEAR (LBF)',4X,'MOMENT (
     1IN-LBF)',5X,'SHEAR (PSI)',4X,'FLEXURE (PSI)'/)
   32 IJKLM=1
      AMOLD=0.
      AKOLD=0.
      DO 3 INDX=2,N2
      I=INDX-2
      AI=PI/64.*(DOU(INDX)**4.-DIN(INDX)**4.)
      AK=PI*(DOU(INDX)**2.-DIN(INDX)**2.)/4.
      AMOM=(YK(INDX+1)-2.*YK(INDX)+YK(INDX-1))*12./DEL/DEL*EMOD*AI
```

```
      UB=5.*PO*CA/(7.*PA*AMACS)
      US=AMACS *CA
      TT=D/US
      AMACB =UB/CB
      ANUB=VISCO (TB,PB,PA,PB)
      REB=UB*D/ANUB
      TDIFR=5.*TT
      IF (IPRNT)2,2,1
    1 WRITE (IO,101) PA,TA,DD,PO,TPLUS,AK
  101 FORMAT ('1'//10X,'TRANSIENT DRAG ON CYLINDERS DUE TO AIR BLAST'//
     110X,'AMBIENT PRESSURE (PSI), PA = ',F16.2/ 10X,'AMBIENT TEMPERATUR
     1E (DEG R), TA = ',F11.2/  10X,'CYLINDER DIAMETER (IN), D = ',F17.3
     1/ 10X,'PEAK OVERPRESSURE (PSI), PO = ',F15.3/ 10X,'POSITIVE DURATI
     1ON (SEC), TPLUS = ',F12.5/ 10X,'DECAY CONSTANT, AK = ',F24.2)
      WRITE (IO,104) CA,US,TT,TDIFR
  104 FORMAT (' '/10X,
     1         'AMBIENT SPEED OF SOUND (FPS), CA = ',F10.2/ 10X,'SPEED OF
     1THE SHOCK FRONT (FPS), US = ',F8.2/  10X,'TIME OF ENGULFMENT (SEC)
     1, TT = ',F14.5/ 10X,'LENGTH OF TIME FOR WHICH THE DIFRACTION PRESS
     1URE APPLIES (SEC), TDIFR = ',F8.5//)
      WRITE (IO,102)
  102 FORMAT (' ',/ 10X,'ALL OF THE FOLLOWING DATA APPLIES AT THE CYLIND
     1ER (STATION C)  '/10X,'UNITS ... PRESSURE IN PSI, TEMPERATURE IN D
     1EG R, VELOCITY IN FT/SEC, TIME IN SEC'//4X,'TIME',6X,'P(OVER)',3X,
     1'TEMP',7X,'Q',8X,'QI',6X,'U',8X,'C',8X,'M',9X,'RE',6X,'P(DIFR)',2X
     1,'P(DRAG)',1X,'P(NET)',1X,'P(NET)PSF'  //)
    2 CON1=8.*(PO*TT)**1.13/(5.*TT)
      DO 3 I=1,ICODE
      IF (ICODE-1)4,4,5
    5 IF (I-1)6,6,7
    6 DELTA=TIME
      TIME=-DELTA
    7 TIME=TIME+DELTA
    4 F=(1.-TIME/TPLUS)*EXP(-AK*TIME/TPLUS)
      DRAG=0.
      IF (TIME-TPLUS)14,14,13
   14 POVER=PO*F
      PC=PA+POVER
      TC=TB*(PC/PB)**.286
      CC=CA*SQRT(TC/TA)
      UC=UB*F*(PB/PC)**.3571
      AMACC =UC/CC
      ANUC=VISCO (TC,PC,PA,PB)
      REC=UC*D/ANUC
      QC=QO*F**2.
      QIC=QIO*F**2.
      PDIFR=0.
      IF (TIME-TDIFR)8,8,9
    8 PDIFR=CON1
    9 CON2=.7
      IF (AMACC -.48)10,10,11
C     NEW DRAG COEFFICIENTS INTRODUCED FEBRUARY 2, 1977.
   10 CON2=.3
      IF (REC-4.E5)12,11,11
   12 CON2=.6
   11 PDRAG=CON2*QIC
      PLOAD=PDIFR+PDRAG
      DRAG=PLOAD*144.
      IF (IPRNT) 3,3,25
   25 WRITE (IO,103) TIME,POVER,TC,QC,QIC,UC,CC,AMACC ,REC,PDIFR,PDRAG,P
     1LOAD,DRAG
  103 FORMAT (' ',1X,F9.5,F9.3,F9.2,2F9.3,2F9.2,F9.3,F11.0,3F9.3,F9.2)
    3 CONTINUE
C     RESTORE DIAMETER D TO ORIGINAL UNITS
   13 D=DD
      RETURN
      END
```

```
    2 WRITE (IO,100) IMODE,FREQ
  100 FORMAT ('1'//10X,'MODE/CURVE NUMBER ',I5
     1                     //10X,'FREQUENCY (CYCLES/SEC)',F12.5//1
     10X,'MOMENTS ARE COMPUTED AT THE GRID POINTS USING 3-POINT OPERATOR
     1S'/10X,'SHEAR FORCES ARE COMPUTED .5*DEL BEHIND THE INDICATED GRID
     1 POINT'//5X,'LOCN',6X,'Y SHAPE (FT)',3X,'SHEAR (LBF)',4X,'MOMENT (
     1IN-LBF)',5X,'SHEAR (PSI)',4X,'FLEXURE (PSI)'/)
      GO TO 32
    1 WRITE (IO,109) IMODE
  109 FORMAT (' '/10X,'TIME (MSEC)',I7//
     1          5X,'LOCN',6X,'Y SHAPE (FT)',3X,'SHEAR (LBF)',4X,'MOMENT (
     1IN-LBF)',5X,'SHEAR (PSI)',4X,'FLEXURE (PSI)'/)
   32 IJKLM=1
      AMOLD=0.
      AKOLD=0.
      DO 3 INDX=2,N2
      I=INDX-2
      AI=PI/64.*(DOU(INDX)**4.-DIN(INDX)**4.)
      AK=PI*(DOU(INDX)**2.-DIN(INDX)**2.)/4.
      AMOM=(YK(INDX+1)-2.*YK(INDX)+YK(INDX-1))*12./DEL/DEL*EMOD*AI
C     NOTE THAT YK IS IN FT, E IS IN PSI, DEL IS IN INCHES, AND THE
C     MOMENT IS IN IN-LBF
      SHEAR=-(AMOM-AMOLD)/DEL
      AKK=(AKOLD+AK)/2.
      AKOLD=AK
      IF (I) 5,5,4
    5 SHEAR=0.
    4 AMOLD=AMOM
      SHSTR=SHEAR/AKK
      FLSTR=AMOM*DOU(INDX)/2./AI
      WRITE (IO,101) I,YK(INDX),SHEAR,AMOM,SHSTR,FLSTR
  101 FORMAT (' ',I7,1X,F15.3,F14.0,F18.0,F16.0,F17.0)
    3 CONTINUE
      IF (ICODE-2) 33,33,34
   33 IJKLM=1
C
C
C     PLOT THE Y CURVE
C
C     ESTABLISH ORIGIN 2 INCHES ABOVE BOTTOM OF PAGE, AND LEAVE SCALES
C     AT 1 INCH = 1 UNIT FOR PLOTTING PURPOSES
      CALL SCALF(1.0,1.0,-2.0,-2.0)
C     CHOOSE THE NUMBER OF INTERVALS ALONG THE X AXIS TO BE AN INTEGER
C     MULTIPLE OF 5 FT.
      CON1=N*DEL/12./5.+.99
      NX=CON1
      CON2=0.
      DO 7 INDX=2,N2
      CON1=YK(INDX)
      CON1=ABS(CON1)
      IF (CON1-CON2) 7,7,8
    8 CON2=CON1
    7 CONTINUE
      CON2=CON2*10.
C     CON2 IS THE RANGE OF ABS(Y) IN TENTHS OF A FOOT
      NY=YLEN*2.
      NNY=NY/2
      NY=2*NNY
C     NY IS THE NUMBER OF Y AXIS INTERVALS (EVEN NUMBER)
      YDEL=CON2/NNY+.99
      IYDEL=YDEL
      IF (YDELL) 30,30,31
   31 IYDEL=YDELL*10.+.99
   30 YDEL=.1*IYDEL
C     IYDEL IS THE YAXIS INTERVAL IN TENTHS OF AN INCH
C     YDEL IS THE Y AXIS INTERVAL IN FEET
```

FORCED VIBRATION (CONT'D)

```
      DX=XLEN/NX
      DY=YLEN/NY
      CALL FPLOT(1,0.,0.)
      CALL FPLOT(-2,0.,0.)
      HTIC=.06
C     PLOTTING THE BOX AND TICK MARKS
      X1=0.
      Y1=0.
      Y2=Y1+HTIC
      DO 11 I=1,NX
      X1=X1+DX
      CALL FPLOT(0,X1,Y1)
      IF (I-NX) 16,11,16
   16 CALL FPLOT(0,X1,Y2)
      CALL FPLOT(0,X1,Y1)
   11 CONTINUE
      X2=X1-HTIC
      DO 12 J=1,NY
      Y1=Y1+DY
      CALL FPLOT(0,X1,Y1)
      IF (J-NY) 17,12,17
   17 CALL FPLOT(0,X2,Y1)
      CALL FPLOT(0,X1,Y1)
   12 CONTINUE
      Y2=Y1-HTIC
      DO 13 I=1,NX
      X1=X1-DX
      CALL FPLOT(0,X1,Y1)
      IF (I-NX) 19,13,19
   19 CALL FPLOT(0,X1,Y2)
      CALL FPLOT(0,X1,Y1)
   13 CONTINUE
      X2=X1+HTIC
      DO 14 J=1,NY
      Y1=Y1-DY
      CALL FPLOT(0,X1,Y1)
      IF (J-NY) 20,14,20
   20 CALL FPLOT(0,X2,Y1)
      CALL FPLOT(0,X1,Y1)
   14 CONTINUE
C     NUMBERING AND LABELING THE AXES
      HT=.11
      Y1=-2.*HT
      XHT=HT*6./7.
      X1=-DX-1.*XHT
      NXX=NX+1
      NXXX=NXX/2+1
      INUM=-5
```

```
      ANUM=(I-1-NNY)*IYDEL*.1
      CALL FCHAR(X1,Y1,XHT,HT,0.)
      WRITE (7,202)  ANUM
  202 FORMAT (F4.1)
   18 CONTINUE
      X1=-5.5*XHT
      NLAB=19
      Y1=YLEN/2.-NLAB/2*XHTT
      CALL FCHAR(X1,Y1,XHTT,HTT,PI/2.)
      WRITE (7,203)
  203 FORMAT ('DISPLACEMENT Y (FT)')
      YSHIF=YLEN/2.
      X1=-DEL
      SCAX=NX*5.*12./XLEN
      SCAY=NY*YDEL/YLEN
      DO 21 INDX=2,N2
      X1=X1+DEL
      XX1=X1/SCAX
      YY1=YK(INDX)/SCAY+YSHIF
      IF (INDX-2) 22,22,23
   22 CALL FPLOT(1,XX1,YY1)
      CALL FPLOT(-2,XX1,YY1)
   23 CALL FPLOT(0,XX1,YY1)
      CALL POINT(1)
   21 CONTINUE
      X1=XLEN+1.
      IF (X1-8.5) 24,24,25
   24 X1=8.5
   25 CALL FPLOT(1,X1,-2.0)
      CALL SCALF(1.0,1.0,0.,0.)
C     ORIGIN HAS NOW BEEN RESTORED TO BOTTOM OF PAGE BEYOND PLOT,
C     READY FOR FUTURE CALLS TO THIS SUBROUTINE
   34 RETURN
      END


      FUNCTION VISCO (TC,PC,PA,PB)
C
C     DYNAMIC VISCOSITY FOR AIR , NU , SEE STN 249
C
      SUM = 90.27754-0.3293413*TC+0.0008741*TC*TC
      VISCO =14.7/PA*(6.+PB/PA)/(1.+6.*PB/PA)
      VISCO =VISCO *((PB/PC)**.714)*SUM*1.E-6
      RETURN
      END
```

```
17    CALL FPLOT(0,X2,Y1)
      CALL FPLOT(0,X1,Y1)
12    CONTINUE
      Y2=Y1-HTIC
      DO 13 I=1,NX
      X1=X1-DX
      CALL FPLOT(0,X1,Y1)
      IF (I-NX) 19,13,19
19    CALL FPLOT(0,X1,Y2)
      CALL FPLOT(0,X1,Y1)
13    CONTINUE
      X2=X1+HTIC
      DO 14 J=1,NY
      Y1=Y1-DY
      CALL FPLOT(0,X1,Y1)
      IF (J-NY) 20,14,20
20    CALL FPLOT(0,X2,Y1)
      CALL FPLOT(0,X1,Y1)
14    CONTINUE
C     NUMBERING AND LABELING THE AXES
      HT=.11
      Y1=-2.*HT
      XHT=HT*6./7.
      X1=-DX-1.*XHT
      NXX=NX+1
      NXXX=NXX/2+1
      INUM=-5
      DO 15 I=1,NXX
      X1=X1+DX
      INUM=INUM+5
      CALL FCHAR(X1,Y1,XHT,HT,0.)
      WRITE (7,200) INUM
200   FORMAT (I2)
15    CONTINUE
      XHTT=1.2*XHT
      HTT=1.2*HT
      X1=XLEN/2.-7.*XHTT
      Y1=-5.*HT
      CALL FCHAR(X1,Y1,XHTT,HTT,0.)
      WRITE (7,201)
201   FORMAT ('POSITION X (FT)')
      X1=0.
      Y1=-10.*HT
      CALL FCHAR(X1,Y1,XHTT,HTT,0.)
      WRITE (7,204) IMODE
204   FORMAT ('MODE/CURVE NUMBER',I5)
      Y1=-DY
      X1=-5.*XHT
      NYY=NY+1
      DO 18 I=1,NYY
      Y1=Y1+DY
```

```
      [illegible]
23    CALL FPLOT(0,XX1,YY1)
      CALL POINT(1)
21    CONTINUE
      X1=XLEN+1.
      IF (X1-8.5) 24,24,25
24    X1=8.5
25    CALL FPLOT(1,X1,-2.0)
      CALL SCALF(1.0,1.0,0.,0.)
C     ORIGIN HAS NOW BEEN RESTORED TO BOTTOM OF PAGE BEYOND PLOT,
C     READY FOR FUTURE CALLS TO THIS SUBROUTINE
34    RETURN
      END


      FUNCTION VISCO (TC,PC,PA,PB)
C
C     DYNAMIC VISCOSITY FOR AIR , NU , SEE STN 249
C
      SUM = 90.27754-0.3293413*TC+0.0008741*TC*TC
      VISCO =14.7/PA*(6.+PB/PA)/(1.+6.*PB/PA)
      VISCO =VISCO *((PB/PC)**.714)*SUM*1.E-6
      RETURN
      END
```

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